ELEMENTS OF MECHANICAL DRAWING

IN TWO PARTS

PART I. FOR BEGINNERS

EXERCISES IN THE USE OF INSTRUMENTS AND USE OF SCALES, AND SIMPLE PROBLEMS IN PROJECTIONS, INTERSECTIONS, AND DEVELOPMENTS

PART II

PROBLEMS IN DESCRIPTIVE GEOMETRY

BY

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PREFACE.

THE aim of this treatise is to supply a course in Mechanical Drawing which shall constitute a satisfactory foundation for all advanced courses.

Part I provides a course for beginners, to acquaint them with the use of drafting instruments and the principles of projections. The instruction is detailed and full to the extent that this course may be pursued not only in colleges and technical schools, but also in secondary schools, and even by the earnest student without a teacher.

Part II comprises problems in Descriptive Geometry to be constructed in the drafting-room to supplement class-room instruction.

The third-angle projection has been adopted in both parts in conformity with the general practice in the drafting departments of business and engineering concerns where practical work is done.

These courses have been taught in Rutgers College with excellent results, and they are believed to supply a satisfactory basis for the higher courses.

I wish to acknowledge my indebtedness to Mr. Richard Morris, Associate Professor of Mathematics and Graphics in Rutgers College, for timely and valuable suggestions.

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RUTGERS COLLEGE, NEW BRUNSWICK, N. J.,
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MECHANICAL DRAWING.

PART I.

CHAPTER I.

DRAWING INSTRUMENTS AND THEIR USES.

1. **Mechanical Drawing, or Drawing with Instruments,** demands exactness as to the direction, form, and magnitude of lines, in contradistinction to free-hand drawing in which these are estimated or determined by the eye. The object of the exercises as far as Plate 10 is to familiarize the student with the uses of drafting instruments, and to teach him to be exact and careful in performing the several operations with them.

2. **List of Materials Required:**

Compasses, 5½ inches, having one fixed leg with a needle-point, the other leg having interchangeable pen and pencil points and a lengthening-bar;

Hair-spring Dividers, 5 inches;

Spring Bow-spacers, about 3 inches;

Spring Bow-pencil, about 3 inches;

Spring Bow-pen, about 3 inches;

Ruling-pen, unhinged, 5 inches, best make.

The instruments thus far mentioned may be purchased in a case together.

Drawing-board, 20 by 26 inches;

T Square, 24-inch blade;

30–60-degree Triangle, the longest side 11 inches.

45-degree Triangle, the side 9 inches;

Scroll, or Irregular Curve (similar to Keuffel & Esser Co.'s No. 26, hard rubber);

Boxwood Flat Scale, 12 inches, graduated in 1, ½, ¼, and ⅛ inch to the foot;

Boxwood Triangular Scale, 12 inches, graduated in 10, 20, 30, 40, 50, and 60 parts to the foot;

Boxwood Rectangular Protractor, 6 by 1¾ inches, with diagonal scales and scale of chords;

Two Drawing-pencils, one 3H, the other 6H;

Pencil-pointer (sandpaper pad, or a sheet of 00 sandpaper);

Ink and Pencil Erasers;

One dozen Thumb-tacks, or a package of *1-oz. Copper Tacks*, for fastening the paper to the drawing-board;

Bottle of Black Water-proof Drawing-ink;

Bottle of Red Water-proof Drawing-ink;

One Ball-pointed Pen (D. Leonardt & Co.);

Two Writing-pens (Gillott's Nos. 303 and 290 respectively);

Pen-holder;

Horn Center; and

Good Drawing-paper as needed.

3. **Care and Uses of Instruments.**—The instruments should be kept free from dried ink and dust, by wiping on a piece of chamois skin or lintless cloth. *Ink should never be allowed to dry in the pen.* The nibs of the pen should be well separated when not in use, and *never screwed together close enough to cause pressure.*

Be careful never to use the edges of the T square or triangles as a guide for the knife to cut the paper, as the edges are thus often cut into and ruined for use as straight-edges. A large pair of shears are most convenient for cutting the sheets of paper into proper sizes. The use of a knife roughens the surface of the drawing-board.

The instruments should be of good quality, although the costliest are not essential for good work. *But with imperfect instruments accurate drawings cannot be made.*

4. **The Compasses.**—These are used for describing arcs of circles, and should always be held so that the jointed legs are nearly perpendicular to the surface of the paper, the fingers lightly clasping the joint, and the compasses slightly inclined in the moving direction. One leg has a fixed needle-point, and to the other leg may be fitted either a pencil-joint or a pen-point; and when arcs of large radii are to be drawn a lengthening-bar may be used to extend this leg. There are joints to both legs of the compasses, so that the lower part may be held perpendicular to the surface of the paper. (This is done in order that the center will not change in describing the arc, and also that both nibs of the pen will touch the paper and thus avoid making a ragged line.) The needle is smoothly pointed at one end, and on the other there is a shoulder; the shoulder prevents the point from entering the paper too far. Just sufficient pressure should be put upon the compasses to keep the point from slipping and still not cause it to penetrate the paper. The pencil in the compasses should be sharpened by wearing away the lead on the outside with the sandpaper, so that it shall taper towards the side next to the other leg of the compasses, leaving that side straight to bring it in close contact with the other leg.

5. **Hair-spring Dividers.** — These are used for setting off measurements on the drawing, but are never used to describe arcs; there is, therefore, no necessity for joints in the legs. A set-screw is adjusted to one leg so that a

slow motion may be given to the movement of one leg to more accurately obtain the measurement to be set off on the drawing.

6. The Spring Bow-spacers, Spring Bow-pen, and Spring Bow-pencil.—These are used, the spacers to space small, equal divisions of a line, and the pen and pencil to draw very small arcs of circles. They will be found very convenient and often necessary.

7. Ruling-pen. — This is used for drawing ink-lines, either straight or curved. It is always guided by a straight-edge, as that of a T square or side of a triangle when drawing straight lines, or a curved edge, as that of a scroll, when drawing curved lines. The pen should be held perpendicular to the surface of the paper, except that it should be inclined slightly, say 5 degrees, in the moving direction. The nibs of the pen should always be held so that they are parallel to the edge, either straight or curved, against which it is held very lightly. The pressure against the edge should be so slight as to avoid shutting the nibs together and thus checking the flow of ink. Pens become dulled from frequent use and need sharpening in order that fine and even lines may be drawn with them; but to be successful in the delicate operation of sharpening the nibs a magnifying-glass should be used to watch the process of wearing away the the surface to a sharp edge on a stone or emery-cloth. The finishing touches must be made on an Arkansas stone of fine grain, or upon emery polishing-cloth (or paper), or one may learn from the instrument-maker the proper way to do it. It is well to have on hand two ruling-pens, so that one may always be in condition for ruling fine lines, and also that when different grades of lines are required in the same drawing one pen may be set for one grade of line and the other for the other grade.

8. The Drawing-board.—Upon this the paper is fastened with the thumb-tacks or 1-oz. copper tacks. It should have a plane, smooth surface. The edges should be straight, so that when the head of the T square slides along and against the edge of the board the blade of the T square will move in parallel positions. Only one edge of the board should be used with the T square during the process of any one drawing; and the board should be so turned as to bring that edge on the left of the draftsman.

9. T Square and Triangles.—While the blade of the T square is used as a guide to the pencil or pen to draw parallel lines which are horizontal, that is, parallel to the top edge of the drawing-board, a triangle may be made to rest against the blade of the T square, and by sliding it along the blade the other side may be used as a guide for drawing parallel lines perpendicular to the edge of the T square. *To draw vertical lines, therefore, do not use the T square against the top or bottom edge of the board, but use one of the triangles resting against the blade of the T square while the head slides against the left edge of the board.* This is important, because the edges of the

drawing-board may not be, and cannot be kept, perpendicular to each other. To draw lines making angles of 30, 45, and 60 degrees with the edge of the T square, with one side of the proper triangle resting against the blade of the T square, use the hypothenuse as a guide for the pencil. By using the two triangles together angles of 15 and 75 degrees may be laid off. Another and important use which may be made of the triangles is to draw parallel lines on the paper in any direction, and also lines perpendicular to each other in any direction. Thus, if the hypothenuse of one triangle be made to coincide with a given line, and an edge of the other triangle (or any straight-edge) be placed against one of the sides of the first triangle, and the first triangle be made to slide along the second, the hypothenuse will move in positions parallel to the given line. By changing the straight-edge to the other side of the first triangle the same system of parallel lines may be transferred to another part of the paper. To draw lines perpendicular to this system of parallel lines, put the other side of the first triangle against the straight-edge and the hypothenuse will then be in a position perpendicular to its first direction. The triangles should be thus used by the student to practise drawing lines parallel and perpendicular to each other until he is familiar with the manipulations.

10. Scroll, or Irregular Curve.—This is used to direct the inking of curved lines which are not arcs of circles. Some part of the edge of the scroll can usually be found to coincide with the curved line on the paper which should be already drawn in pencil, and after allowing a sufficient space between the edge of the scroll and the line for the pen to move over the line while resting against the scroll, this part is inked in; the scroll is then moved until some other part is found to coincide with a continuous part of the pencilled line and that part also inked; continuing thus the pencilled line is completely inked. Great care should be observed to turn the pen as it moves along the edge of the scroll so that the nibs shall remain tangent to the edge, otherwise the ink will run out of the pen on the scroll and produce a blot on the paper.

11. Boxwood Flat Scale.—This scale is graduated on the edges by lines that are one inch apart in one case, and different fractions of an inch in other cases, and the space at one end of the graduation is divided into 12 equal parts. For example, in one case the space is one-half an inch laid off successively along the edge, and the end space is divided as stated above. If the scale of the drawing is ½ an inch to 1 foot, that is, one foot on the object to be represented is made ½ inch on the drawing, then this scale may be used directly to scale off distances on the drawing representing the corresponding feet and inches on the object.

12. The Triangular Scale.—This is a scale of equal parts, that is, the six different faces are divided so that

on one and another there are 10, 20, 30, 40, 50, and 60 divisions to each inch, respectively. This is called an engineer's scale, and is generally used where long distances are represented by proportionately short distances on the drawing. For example, if it is desired to represent on the drawing by one inch a distance of 60 feet, the scale of 60 parts to the inch would be used where each division of the scale would represent one foot.

13. **The Protractor.**—This is used to measure angles between lines on a drawing, and also to lay off from one line at a given point an angle another line is to make with the first line. In this particular form of protractor three of the edges are divided in degrees from zero to 180, the middle of the fourth side being the center of the radial lines marking the degrees. It also contains a scale of chords upon which are shown the chords of each degree from zero to 90 on an arc whose radius is the chord of 60 degrees on the scale. This scale is also used to measure and lay out angles. On the reverse side is a diagonal scale which divides the inch (or half-inch) into tenths and hundredths. The construction of the diagonal scale and a scale of chords are exercises to be done by the student later, when further explanation will be given.

14. **Pencils.**—The 3H pencil should be sharpened on one end to a long, sharp, rounded point for marking letters and figures. The 6H pencil should be sharpened on one end as is the 3H, but on the other to a long, flat, chisel-shaped edge, with the corners slightly rounded, thus:

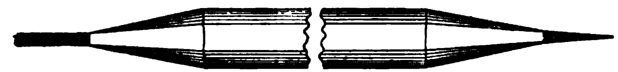


FIG. 1.

In drawing a line through a point the straight-edge (T square or triangle) should be brought to half cover the point and the pencil held vertical with the flat side of the chisel-edge pressed against the straight-edge; then if a line is drawn it will pass through the point. If the round point is used instead, allowance must be made by placing the straight-edge some distance from the point, so that when the pencil is held vertical it will pass through the point, otherwise if the pencil-point is inclined toward the straight-edge to make the pencil pass through the point, it is likely to run under the edge where the ruler is not held firmly against the paper and a straight line is not drawn. The points of the pencil should be kept very sharp by cutting away the wood from the lead and wearing the lead to the proper shape on the sandpaper pencil-pointer and afterwards polishing the lead off on a piece of cloth.

15. **Ink.**—The best ink is made by rubbing the China or India stick ink in a small quantity of water, on a porcelain slab, until a sufficient quantity has been worn off to make the ink of the desired consistency when mixed

with water. This, however, requires considerable time, and, except for very fine drawings, it is hardly worth while when there are inks of good quality on the market already mixed and bottled. The best of these answer for all ordinary drawings. None of the writing-inks should ever be used in the pens of the drawing-instruments, as the acids in these inks quickly corrode and roughen the edges of the nibs, making the pens useless. Colored inks are made from artist's water-colors, or they are found in the market, as is the black ink, already mixed and bottled. The ordinary colored inks made for writing purpose must be avoided. India ink rests on the surface of the paper and may easily be erased. It does not penetrate the paper as do the chemical inks.

16. The Writing-pens. — These are for making letters and figures on the drawings.

17. Horn Center.—This is a small disc of thin, transparent horn with three fine, sharp points to hold it in place on the paper, and is used to place over the center of several concentric circles, so that the frequent placing of the compass-point on the center will not make a large hole and destroy the good appearance and accuracy of the drawing.

18. Drawing-paper. — Whatman's drawing-paper, the hot-pressed, which has a smooth surface, or any other good drawing-paper, may be used for the drawings of this course. If Whatman's paper is used, that known as "Medium" 17″ by 22″ makes two plates the size to be used in this course.

CHAPTER II.

GENERAL INSTRUCTIONS AND PRACTICE EXERCISES WITH INSTRUMENTS.

19. The series of drawings following are to be made on plates of uniform size, 14 inches long by 9.5 inches wide. A border-line extends around the plate, one-half an inch from the outside edge, except on the left end, where it is one inch from the edge to allow for binding. The dimensions inside of the border-line, therefore, will be 12.5 inches by 8.5 inches. Some plates will be sub-divided, in pencil only, into six equal squares, with sides 3.5 inches, separated from each other and from the border-line by a space of one-half an inch, as shown in Diagram A. The squares are numbered from left to right, beginning in the upper row. The *initial point* or *origin* of a square is the lower, left-hand corner, the *reference axes* being the lower and left sides of the square. x represents the *horizontal*, and y the *vertical*, *ordinate* of any point. For example, if the point is 2 inches from the left-hand axis and 1.5 inches from the bottom axis, the point is designated thus: $x=2''$, $y=1.5''$, or simply $(2'', 1.5'')$, the x distance being always placed first. When the problem or drawing requires the whole plate the squares are omitted and the left-hand and bottom border-lines will be the reference axes.

The word *Plate* and the *number of the plate* are to be printed at the upper right-hand corner, midway between the border-line and the outside edge, and the *student's name* in the lower right-hand corner in a corresponding position. The height of the letters is to be $\frac{1}{8}$ of an inch, no part of the printing to extend to the right of the right-hand border-line.

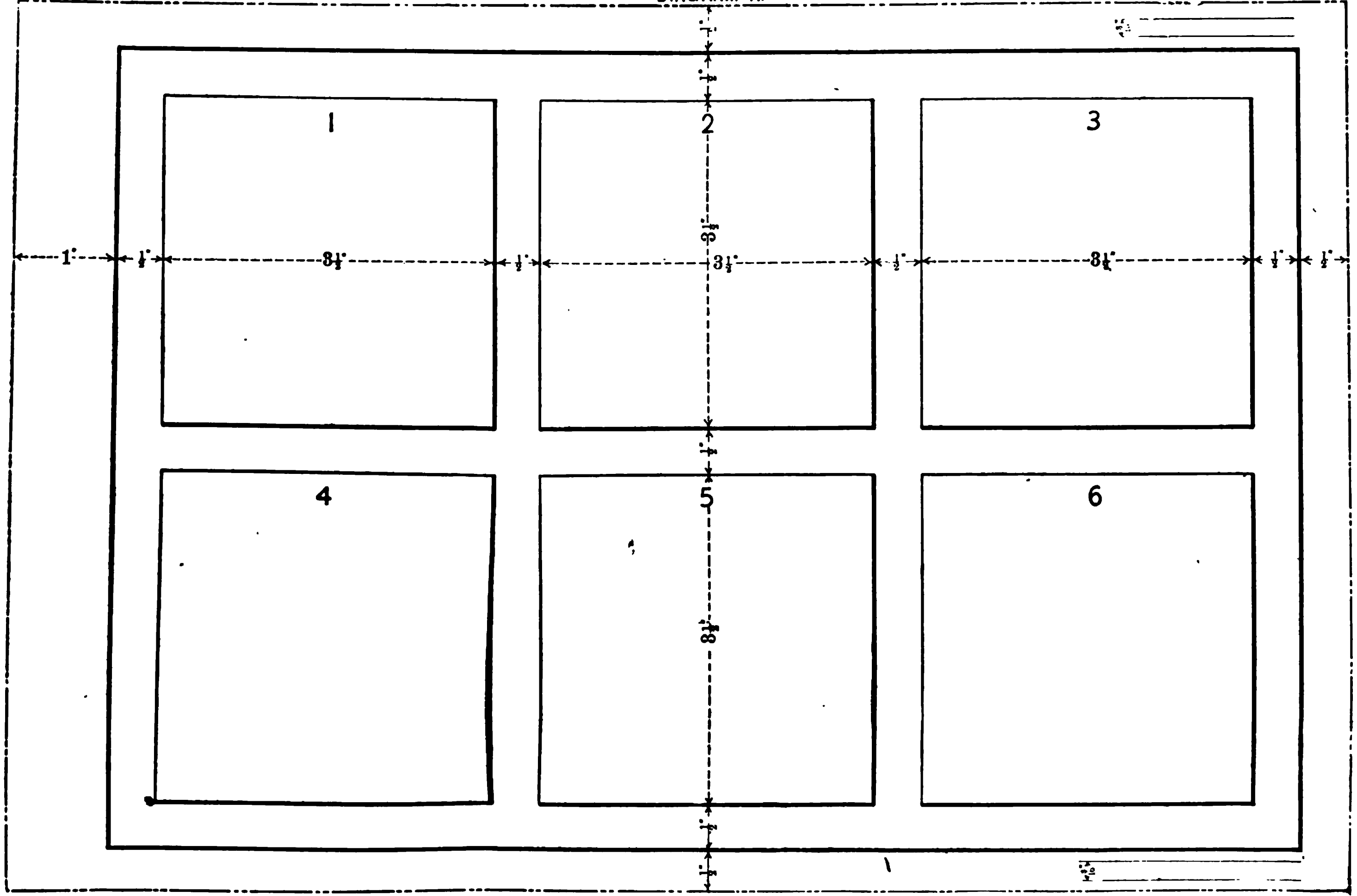
20. A line that is parallel to the bottom axis of the plate will be known as a *horizontal line*, or simply *a horizontal*, and a perpendicular to the horizontal line will be known as a *vertical line*, or simply *a vertical*.

21. Plain, or Gothic, Letters and Numerals.—The correct formation of letters and their proper spacing can be learned only by a careful study of details, and practice alone will make the student an adept at lettering.* But for the letters to be made in the following series of drawings the simple style of letter shown in Plate A will be used.

A few general principles will be noted in regard to this style of letter.

* See "Lettering," by C. E. Sherman, and "Lettering for Draftsmen, Engineers, and Students," by C. W. Reinhardt.

DIAGRAM A.



a. Capital letters and numerals which go together should be the ***same height***, and the small letter (lower case) corresponding should be ***three-fifths the height of the capital.***

b. The ***breadth*** of letters in proportion to their height varies but a good proportion is as follows: Calling one-fifth of the height of the capital letter ***a unit***, then the breadth of

A, C, G, K, O, Q, X, Y	$= 4\frac{2}{3}$	units,
D, T, V, Z	$= 4\frac{1}{4}$	"
B, E, R, S	$= 4$	"
F, H, N, P, U	$= 3\frac{3}{4}$	"
L	$= 3\frac{1}{2}$	"
J	$= 3\frac{1}{3}$	"
M	$= 5$	"
W	$= 6\frac{1}{4}$	" and
I	= a single line.	

c. For the small letters the breadth of the body part should be a shade narrower than its height, that is, a shade less than three units, and for those letters which extend above or below the body part the extension is two units except in the *t*, which extends above one unit only.

d. In the inclined letters the slant is 3 horizontal to 8 vertical. The length of the stems is the same as in the upright letters, making the vertical height a little less. The breadth of inclined letters is governed by the rule which applies to upright letters.

e. **Spacing.**—The general principle to be observed in spacing is to make the letters of a word ***appear*** to be a uniform distance apart. If the distances of the letters apart are equal, the effect will be, in general, to make them appear to the eye unequally separated, because when the letter are placed side by side their various forms in different combinations give varying areas of space. To illustrate: In the word WAVERLY, if the letters are equally spaced, thus, W A V E R L Y, they do not appear to be, but when unequally spaced thus, WAVERLY, the spacing is correct. Again, in the word INMATE, if equally spaced thus, I N M A T E, the appearance to the eye is not symmetrical, but when printed thus, INMATE, the spacing being unequal, still the appearance is correct. A little examination of these and other examples will lead to the general conclusion that if the areas between the letters appear to the eye to be as nearly equal as possible the spacing will be correct. Where two letters with straight, vertical stems, as the I and N, or the N and M, in the word INMATE come together, the distance must be increased, and when such letters as the L and Y, with inclined stems, come together, as in the word WAVERLY, the distance must be lessened. It would be ***impracticable*** to attempt to

make rules governing the multitude of permutations and combinations of letters that are likely to come together in printing words, and so, aside from the general conclusion reached above, the student must depend upon his eye to determine the proper spacing of letters.

f. Two horizontal guide-lines should always be drawn, in pencil, to mark the tops and bottoms of the letters. Letters with curved stems, as C, G, O, Q, S, should extend a shade above and below the guide-lines; while letters bounded by horizontal lines at top and bottom, or either, as B, D, E, F, L, should be a little scant, if anything, in order that the two kinds of letters coming together may appear to be of the same height. Letters terminating in vertical or slant lines at top or bottom, or both, as F, H, I, K, etc., or in a sharp angle, as A, M, W, etc., should extend a shade beyond the guide-line for the same reason. Letters B, E, H, S, and Z should be made larger at the bottom than at the top in order that the symmetrical parts should *appear* to be equal. If the student will invert such letters, when correctly printed, he will observe how much the bottom is larger than the top.

22. Copy the letters of Plate A, after reading carefully Art. 21.

The lower guide-line of the upper row is at $y=6.9$ inches.

The lower guide-line of the second row is at $y=5.75$ inches.

The lower guide-line of the third row is at $y=4.8$ inches.

There should be four guide-lines for the small letters: one guide-line to mark the bottom of the body part of the letter, a line three units above to mark the height of the body part, a line two units below, and another line five units above; the highest and lowest lines marking the extremities of such letters as *b* and *p* above and below the body part.

The lower guide-line of the fourth row is at $y=3.7$ inches.

The lower guide-line of the fifth row is at $y=2.6$ inches.

The lower guide-line of the sixth row is at $y=1.6$ inches.

The height of the capital letters and the numerals is *three-sixteenths of an inch;* the unit is, therefore, one-fifth of this distance. The height of the body part of the small letters is three units. With the spring bow-spacers divide three-sixteenths of an inch into five equal spaces, then with this space unchanged on the instrument it may be used to measure the widths of the letters as given in Art. 21. The vertical height of the slant capitals and numerals is obtained by laying off on the proper slant five units and drawing the upper guide-line through the extremity of the distance thus laid off. Similarly the guide-lines for the small letters may be obtained.

23. **Directions.**—(*a*) Use the *ball-point pen* for drawing the capital letters and numerals, and the 303 *Gillott pen* for the small letters.

b. Learn to make the letters *free-hand*, that is, without the use of the ruling-pen and straight-edge, and with a *steady, single stroke, without sketching.*

c. In drawing stra'ght stems the stroke should be a *pulling* stroke, and never a *pushing* one. To draw the straight horizontal strokes, turn the drawing-board or change your position in order to draw the pen toward you.

d. In drawing *curved letters*, as the O for instance, make it in two strokes, joining them at top and bottom, thus: . The arrows show the direction in which the pen should move. The illustration shows a separation at top and bottom to indicate where each stroke is to begin and end; the student should join these carefully in his work.

e. To make the curved letters *symmetrical* they should be "blocked in," that is, they should be circumscribed by a rectangle for the upright letters, and by a parallelogram for the slant letters, and the curves made tangent to the sides.

f. In drawing the *straight, horizontal,* and *vertical strokes* take great pains to make them respectively exactly horizontal and vertical. A very slight variation of the top of a vertical stroke to the right makes it look tipped, while a slight variation the other way is not noticeable. In fact a slight inclination of the top to the left is to be recommended.

24. **Directions about Drawing Lines.**—To draw *horizontal lines*, the edge of the T square guides the motion of the pencil and ruling-pen, and to draw *vertical lines* the side of a triangle, when the other side rests against the edge of the T square. Lines making *angles of 30, 45, and 60 degrees* with a horizontal are drawn by

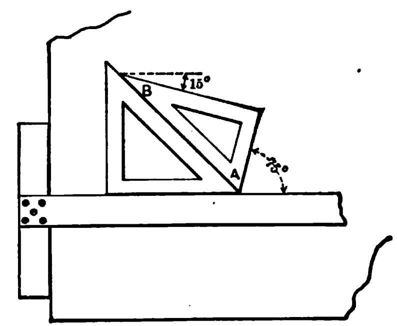


FIG. 2.

placing one side of the proper triangle against the T square and using the hypothenuse as a guide for the pencil. To draw lines making *angles of 15 and 75 degrees* with a horizontal, the triangles are placed as in Fig. 2.

If the line is to make *15* or *75 degrees* with the horizontal on the other side, interchange the positions at the angles *A* and *B* of the outer triangle.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

1234567890

abcdefghijklmnopqrstuvwxyz

ABCDEFGHIJKLMNOPQRSTUVWXYZ

1234567890

abcdefghijklmnopqrstuvwxyz

To draw a line or a series of lines *parallel* or *perpendicular to any oblique line*, the triangles may be used as explained in Art. 10.

25. Lines will be thus designated:

Full lines (unbroken or continuous)

Broken lines (dashes)

Dotted

Broken and dotted

or

Lines may be ***heavy***, ***medium***, or ***light*** in their relations to each other. What may be considered a light line in one drawing as compared with other lines in the same drawing, in another drawing may be considered medium or heavy. The lines below are light, medium, and heavy, respectively, with relation to each other.

Care should be taken to make all heavy, medium, and light lines of uniform grade or thickness, respectively, on the same drawing.

"Broken lines" should have the dashes the same length, about $\frac{1}{8}''$ long, and the spaces equal, about one-third the length of the dash.

Dotted lines are made by holding the drawing-pen vertical and merely touching the paper. If the pen is properly sharpened the mark made on the paper will then be a slightly elongated dot. The open space and the dot should be equal in length.

PLATE 1.

26. Exercise for Practice in the Use of the Ruling-pen.—Lay out the lines of the plate in pencil as in Diagram A, after fastening the paper to the drawing-board with the thumb-tacks or 1-oz. copper tacks. Measure distances consecutively from one line with the boxwood flat scale. Use the T square as a guide for the pencil in drawing horizontal lines, and one of the triangles for vertical lines.

In squares 1, 3, and 5, with the spring bow-spacers, *by trial*, divide the left side of each into eight equal spaces, adjusting the screw of the spacers until the points are the exact distance apart. Then with the *chisel-edge* of the pencil draw horizontal lines through the points of division from the left to the right sides of the square. *Draw very light lines with the pencil*, as all pencilled lines are to be either covered by ink or erased.

In square 2 divide the top side of the square into eight equal spaces, and draw lines through the points of division, using the side of the triangle as a guide for the pencil, while the other side of the triangle slides along the edge of the T square.

In squares 4 and 6 draw lightly, *in pencil only*, a diagonal of each square perpendicular to the direction in which the lines are to be drawn, then divide each diagonal into ten equal parts, and through these points of division, with the hypothenuse of the 45-degree triangle as a guide, draw lines limited by the sides of the square.

After drawing all these lines in pencil they are to be drawn in ink. Place ink between the nibs of the ruling-pen by means of the ink-filler in the stopper of the bottle, or with an ordinary writing-pen, so that there will be some pressure of the ink towards the point, but not enough to cause the ink to drop out by a slight jerk of the hand. Have a waste piece of paper to try the pen on, and adjust the set-screw in the pen to the proper width to produce a line of the required thickness. Follow the copy in Plate I as to the kinds of lines to be drawn. Observe directions in Art. 7 as to the use of the ruling-pen, and draw the lines in ink over the pencil-lines, starting exactly on the line of the side of the square, moving the pen to the right and stopping exactly on the opposite side. In the horizontal lines move the pen from left to right, in vertical lines from bottom to top, in the diagonal lines of square 4 from the bottom towards the top, and in square 6 from the top towards the bottom.

Draw the border-lines and print the word PLATE and its number and your name as directed in Art. 19.

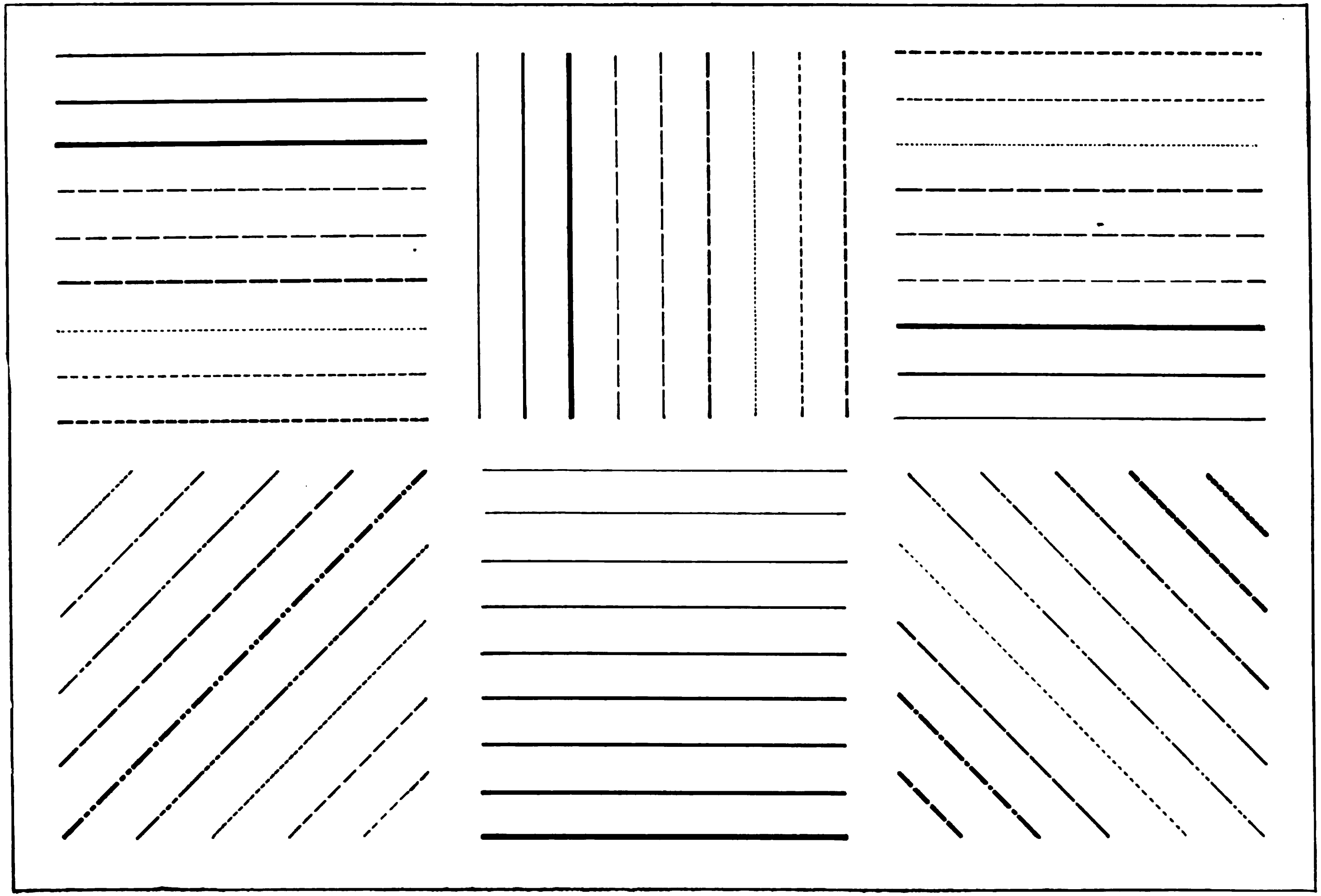


PLATE 2.

27. Exercise in Drawing Long Lines with the Ruling-pen.—Draw a line, in pencil only, on each side and end of the plate, one-half an inch inside of the border-line. Divide the line on the left side into twenty equal spaces, thus: Use the hair-spring dividers to bisect the distance between the upper and lower lines, and to bisect the two equal spaces thus formed, then with the spring bow-spacers divide each of the four remaining spaces into five equal spaces. Draw horizontal lines in pencil through each of the points of division. Afterwards draw over the pencil-lines ink-lines, beginning with the upper line for the first line, and ending with the bottom line for the twenty-first, as follows:

Three full, light lines (see Art. 25).
Three full, medium lines,
Three full, heavy lines,
Three broken, light lines,
Three broken, medium lines,
Three broken, heavy lines,
One dotted, light line,
One dotted, medium line, and
One dotted, heavy line.

Begin exactly on the pencil-line one-half an inch inside of the left-hand border-line, ***hold the pen at the same angle*** (see Art. 7) ***from start to finish***, and end exactly on the pencil-line one-half an inch inside the right-hand border-line. After inking all lines erase with a soft pencil-eraser all pencil-lines which show, ink in the border-lines, and print the word Plate, its number and your name, as before.

PLATE 3.

28. Exercise in Drawing Arcs and Circles with the Compasses.—Divide the plate into six equal squares as in Diagram A. In square 1, find the center by drawing, in pencil only, the diagonals of the square. Through the center draw a horizontal line and divide it into sixteen equal spaces, or one-half of it into eight. Draw in pencil with the compasses concentric circles passing through the points of division, the center of the square being the common center of the circles. Use the spring bow-pencil for the two smaller circles. In order to avoid gouging a hole in the paper in drawing several circles with the same center, use the horn center and let the needle-point of the compasses rest on it. (Observe carefully the instructions in Art. 5.)

In square 2, find the enter of the square as before and divide the lower half of the vertical line through the center into eight equal parts, and through the points of division draw circles whose radii vary in length consecutively from one to eight spaces. The centers of the respective circles fall on the points of division, consecutively, from bottom to middle. *Be sure that all the circles are exactly tangent to each other at the bottom.*

In square 3, draw a horizontal line through the center of the square and divide it into sixteen equal spaces. With these points of division as centers, consecutively, and radii varying in length from one space to eight spaces, draw semicircles, beginning at the left, below the horizontal line, then with radii varying consecutively from eight spaces to one space draw the remaining eight semicircles above the horizontal line. *Be careful that the circles are exactly tangent where they meet on the horizontal line, and also on the outside.*

In square 4, find the center of the square and draw a circle tangent to the four sides of the square. Draw horizontal and vertical lines through the center, and the other radial lines, using the 30–60-degree triangle. By trial find a circle, its center on the vertical line, which shall be tangent to the outside circle and to the two adjacent radial lines, or this may be done by inscribing a circle in the equilateral triangle formed by the two adjacent radial lines and the side of the square. Through this center on the vertical line draw a circle whose center is at the center of the square. This circle will cut all the other radial lines in the centers of the corresponding circles. With these centers draw the smaller circles, as

shown in the plate, tangent to the outside circle and to each other.

In square 5, divide the left and bottom sides of the square each into four equal spaces and draw, in pencil only, horizontal and vertical lines through the points of division. The intersections of these lines are the centers of the respective circles. The lengths of the radii of the circles are equal to a half and a whole space respectively. *Be very careful to determine the exact positions of the centers of the circles,* so that the circles will be tangent and not intersect.

In square 6, find the center of the square and draw a circle tangent to the sides. Draw another circle with the radius one-sixteenth of an inch less. Draw horizontal and vertical lines through the center of the square, and the other radial lines by using the 45-degree triangle. Draw a circle with a radius one and one-sixteenth inches, its center at the center of the square, cutting the radial lines in the centers of the smaller circles, respectively, which are drawn tangent to the inner of the two outside circles. Three other concentric circles are then drawn with radii of one-half, seven-eighths, and fifteen-sixteenths of an inch respectively.

After these exercises are done in pencil they are then to be drawn in ink. Follow the copy in the thickness of the lines. Use the compasses for the larger circles and the spring bow-pen for the smaller. (Observe carefully the instructions in Art. 5.) Use the horn center where several concentric circles are to be drawn to avoid inaccuracy caused by the needle-point of the compasses wearing a hole in the paper at the center, and allowing the point to shift its position as the circle is drawn.

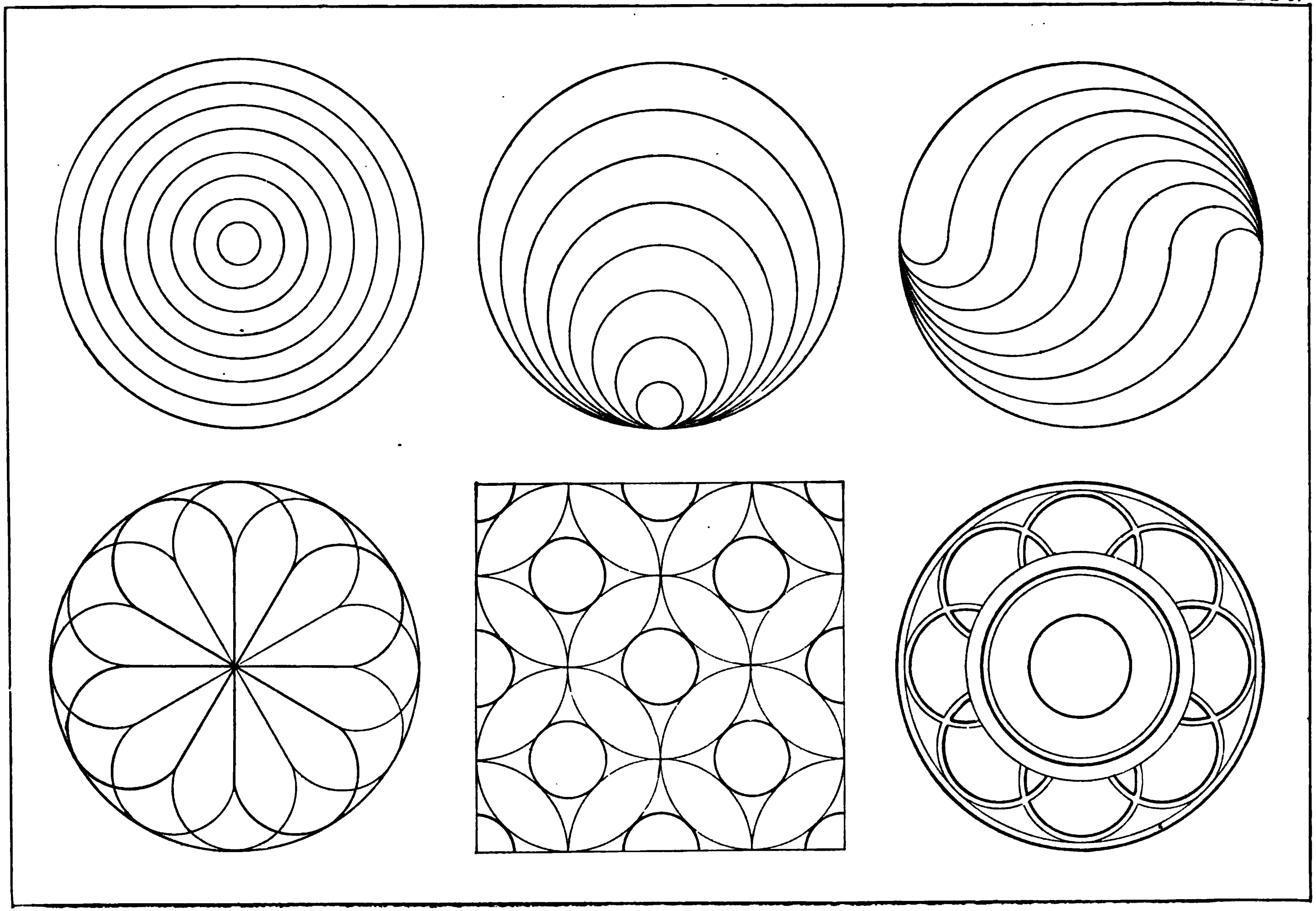


PLATE 4.

29. Exercise in Drawing Simple Designs, Straight Lines Tangent to Curves, and in the Use of the Irregular Curve.— In square 1, divide two opposite sides into eight equal spaces and through the point of division nearest one corner, and each alternate point, draw, first in pencil and afterwards in ink, one set of parallel lines. After drawing lines through the alternate points on one side continue through the corresponding points on the opposite side. Or one side may be divided, as stated, with the bow-spacers and the corresponding points found on the opposite side by transferring them across by the use of the T square.

Use the 45-degree triangle, one side resting against the edge of the T square, and the hypothenuse as a guide for the pencil. Reverse the triangle to draw parallel, diagonal lines drawn in the other direction.

In square 2, divide the sides as before and draw parallel, diagonal lines through each point of division. Ink in also the heavier horizontal and vertical lines as shown.

In square 3, divide two adjacent sides into eight equal spaces, and through the points of division draw, in pencil only, the respective horizontal and vertical lines (Art. 9) as shown in Fig. 3. The diagonal lines, making an angle of 45 degrees with a horizontal, are to be drawn as shown in the figure, first in pencil, then in ink.

In square 4, draw the horizontal and vertical lines, in pencil only, as in Fig. 3. The small circles in Fig. 4 enclose the centers of the several circles or arcs which are shown. Draw the arcs first and afterwards the straight lines tangent to the arcs. These should be drawn in pencil first, then in ink.

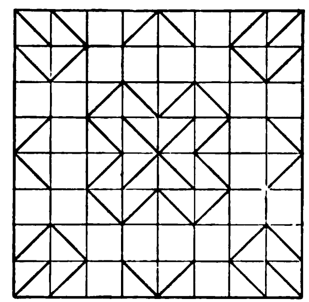


FIG. 3.

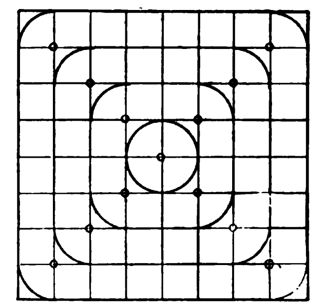


FIG. 4.

In square 5, divide two adjacent sides of the square into sixteen equal spaces. Draw, in pencil only, horizontal and vertical lines through the points of division as shown in Fig. 5. The smallest circles mark the centers

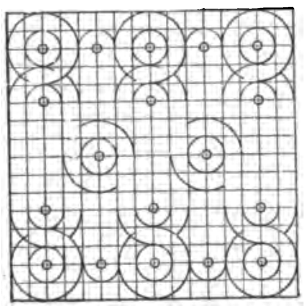


FIG. 5.

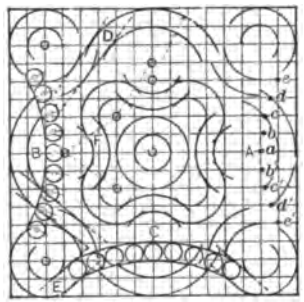


FIG. 6.

of the respective circles, which have radii equal in length to one space and two spaces respectively. A study of Fig. 5 will make clear which lines are to be inked and which erased. Draw all arcs first and the straight lines tangent to the arcs afterwards.

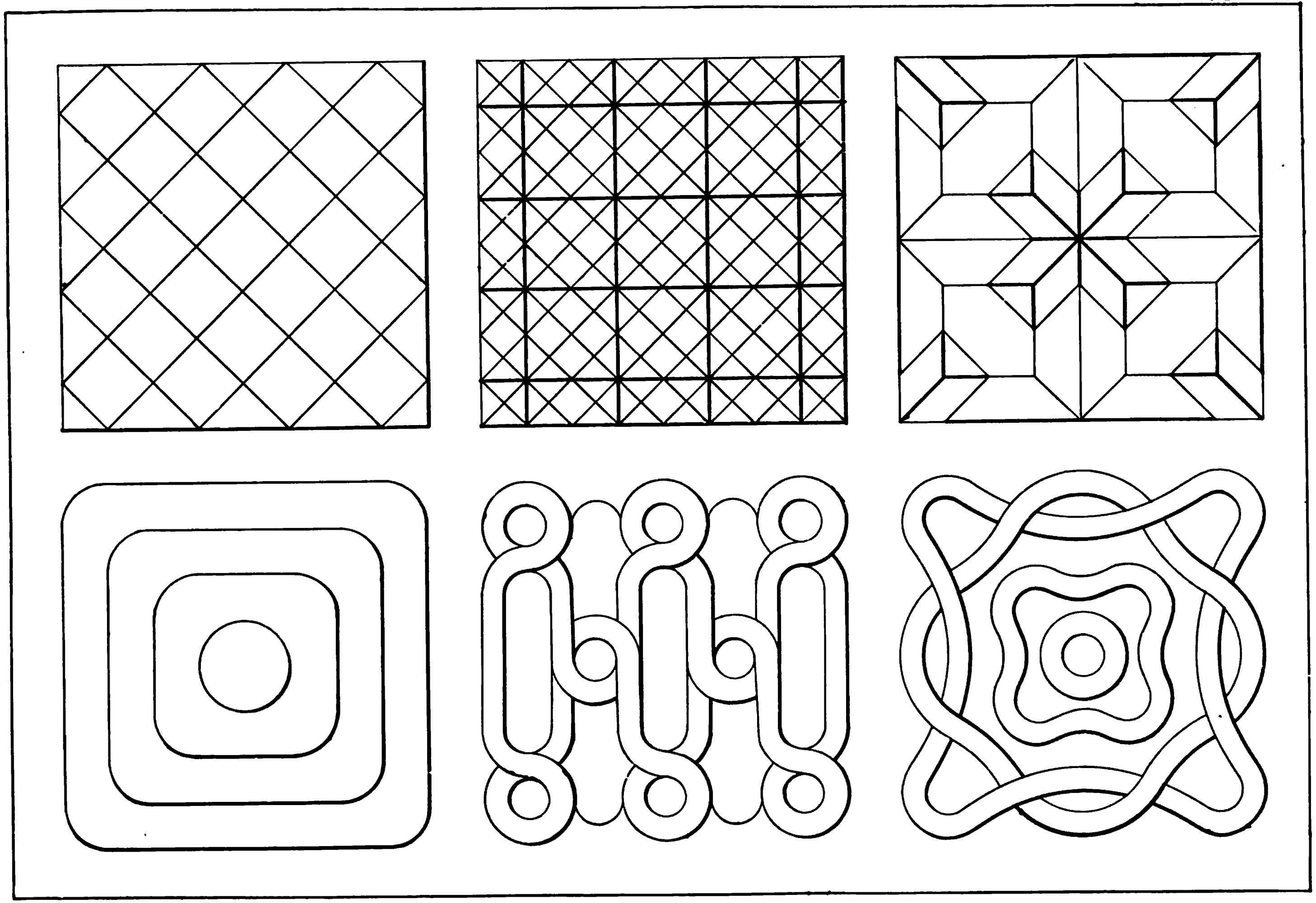
In square 6, draw horizontal and vertical lines as in Fig. 5. Study carefully Fig. 6.

At A, Fig. 6, the points *a*, *b*, *b′*, *c*, *c′*, *d*, *d′*, *e*, and *e′* are located as follows: *e* is at the intersection of the first vertical line from the right side with the fourth horizontal line from the top side; *d* is on the fifth horizontal line and bisects the distance between the first and second vertical lines; *c* is on the sixth horizontal line and bisects the remaining distance, or is three-fourths the distance from the first to the second vertical line; *b* is on the seventh horizontal line and bisects the remaining distance, or is seven-eighths the distance from the first to the second vertical line; and *a* is at the intersection of the eighth horizontal with the second vertical line. *e′*, *d′*, *c′*, *b′* have corresponding positions, respectively, on the other side of the middle, or eighth, horizontal line. Corresponding points are found on all four sides of the square. Through these points, with the irregular curve as a guide to the pencil, a continuous line is drawn. If no part of the edge of the irregular curve will coincide with all the points at the same time, then different parts of the curve must be made to coincide with at least three points at the same time, and care must be exercised to have the ending of one part of the curved line and the beginning of another part continuous and tangent to the same straight line at the point of junction (see Art. 10). At B the next step is shown. Small circles are drawn with radii equal to one-half a space, tangent to the irregular curve at the points *a*, *b*, *e*, etc., that is, the radii are respectively normal to the curve at these points. At C the next step is shown. The inner curve is drawn, with the irregular curve as a guide to the pencil, tangent to these small circles.

The arcs at the corners and sides of the square, with their respective centers and lengths of radii of one or

more spaces, as is shown in the figure, are drawn as indicated. Straight lines tangent to each of the two curves which they join are drawn as shown at D and E. The design in the middle of the square is made up of arcs of circles whose centers are shown in the figure. The lengths of the radii are always one or more spaces, except at F and the three other corresponding positions, where the center comes at the intersection of a horizontal and a vertical line, and the length of the radius is such as to make the arcs tangent on the line joining their centers.

The arcs are drawn in pencil beyond the tangent points as shown in the figure, but when the inking is done *great care should be taken to end each arc or straight line exactly at the tangent point of junction.*



CHAPTER III.

GEOMETRIC CONSTRUCTIONS, AND CONSTRUCTION AND USE OF SCALES.

30. **Geometric Constructions.** — In constructing the following problems the *number of the problem* is to be printed at the top of the square, midway between the sides, the top of the numeral coinciding with the upper line of the square, and its height to be three-sixteenths of an inch.

All *given lines* in the problems are to be drawn *light, full, in black ink; required lines, medium, full, in black ink; construction lines, light, full, in red ink;* and *explanation and projecting lines, light, dotted in red ink.* (Lines as shown in the accompanying plates when broken represent construction lines, and when dotted represent explanation or projecting lines.) When a point is given draw a very small circle around it in red ink, and when required the same in black ink.

Foot and *feet* will be represented by one accent, and *inch* and *inches* by two accents; thus, 12 feet 8 inches is represented by 12′ 8″.

The problems are to be constructed by the use of the *compasses*, a *straight-edge*, and *scales*. Nevertheless when either horizontal, vertical, or oblique lines are *given lines* the *T square* and *triangles* may be used in drawing them.

The problems are to be constructed within the squares in consecutive order.

PLATE 5.

Prob. 1. Bisect a given horizontal line $2\frac{1}{2}''$ long by a required vertical line $2''$ long. For the left end of the horizontal line $x=\frac{1}{2}''$, $y=1\frac{3}{4}''$.

Prob. 2. Given a horizontal line the same as in Prob. 1.
(a) At the left end of this line erect a perpendicular $1\frac{1}{2}''$ long.
(b) At $2''$ from the left end let fall a perpendicular $1''$ long.

Prob. 3. Given a horizontal line as in Prob. 1; also given a point $x=3''$, $y=2\frac{3}{4}''$.
(a) Let fall a perpendicular from the point to the line.
(b) Draw a line $1''$ long parallel to the given line from a point $(1\frac{1}{4}'', \frac{3}{4}'')$.

Prob. 4. Draw a line making an angle of 30° with the horizontal, $3''$ long, from the point $(\frac{1}{2}'', 1'')$; divide this line into 12 equal parts, using a horizontal line as an auxiliary line, and using the two triangles to draw parallel lines (Art. 9).

Prob. 5. With the aid of the two triangles lay off at the point $(\frac{1}{2}'', \frac{1}{2}'')$, on a horizontal line, angles of 15°, 30°, 45°, 60°, 75°, and 90°, making the radial lines terminate in the sides of a $2\frac{1}{2}''$ square, with the given point a corner.

Prob. 6. (a) Given an angle of 60° laid off from a horizontal at the point $(\frac{1}{2}'', \frac{1}{2}'')$; bisect the angle.
(b) Given an angle of 90° laid off from a horizontal at the point $(3'', 3'')$; trisect the angle.

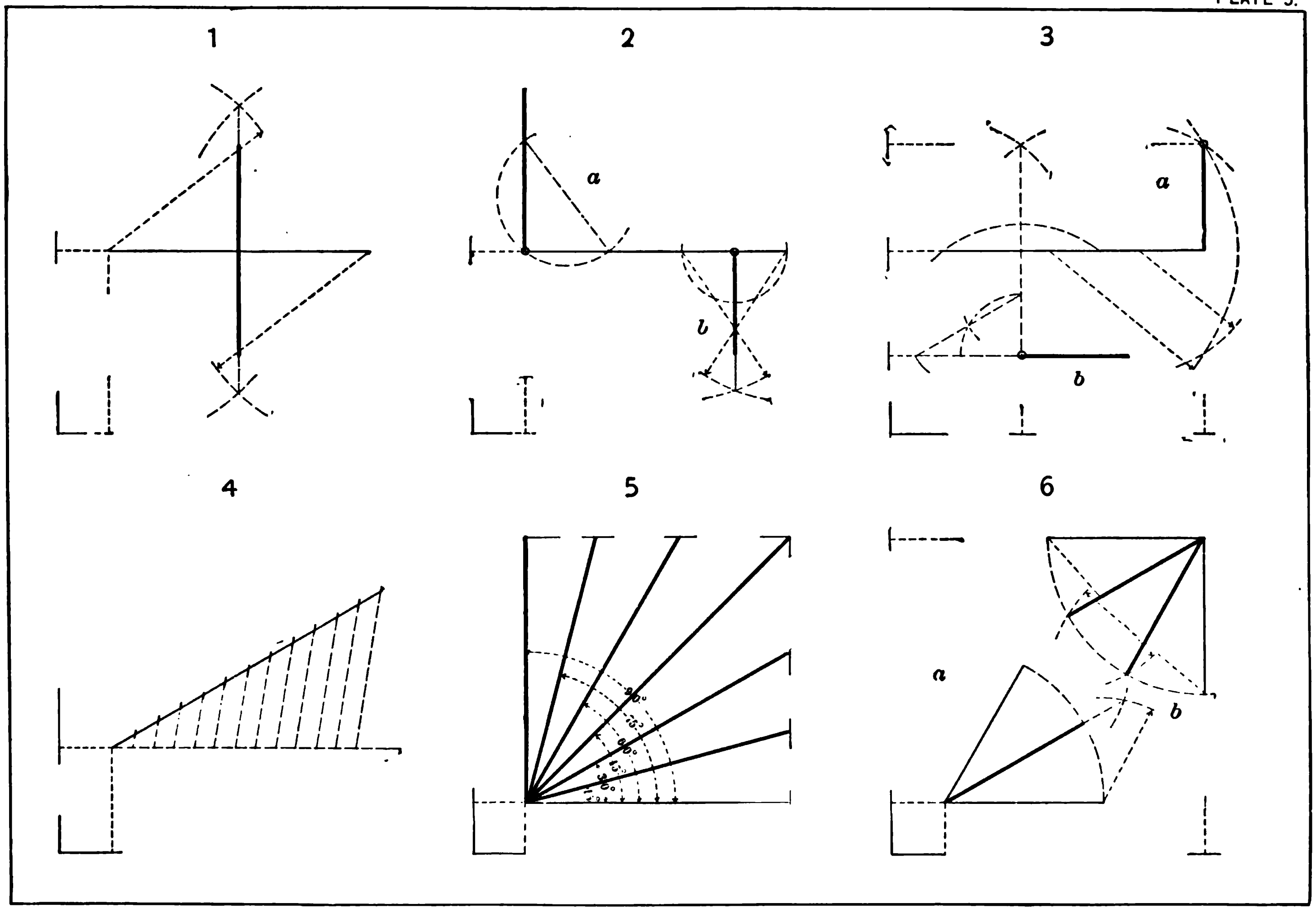

1
2
a
b
3
a
b
4
5
6
a
b

PLATE 6.

Prob. 7. With a radius of $1\frac{1}{2}''$ draw an arc through the two points $(1\frac{1}{4}'', 1'')$ and $(\frac{1}{2}'', 2\frac{1}{2}'')$.

Prob. 8. Draw an arc through the three given points $(\frac{3}{4}'', 2'')$, $(1\frac{3}{4}'', 3'')$, and $(3'', \frac{3}{4}'')$.

Prob. 9. With a radius of $1''$ and center at $(1\frac{1}{4}'', 1\frac{1}{4}'')$ draw a circle. From a given point $(3'', 3'')$ draw two tangents to the circle.

(Suggestion: Bisect the line joining the given point and the center of the circle, and with this point of bisection as a center and a radius equal to half the length of the line describe an arc: this arc will cut the given circle in the points of tangency.)

Prob. 10. With a radius of $1''$ and center at $(1\frac{1}{4}'', 1\frac{1}{4}'')$ draw a circle, and with a radius of $\frac{1}{2}''$ and center at $(2\frac{3}{4}'', 2\frac{3}{4}'')$ draw another circle. Draw two tangents, one of which will cut the line of centers between the circles and the other outside, if produced. Find the points of tangency and join them. (Suggestion: For the first tangent draw a circle concentric with the larger circle and with a radius equal to the difference between the radii of the two given circles; next draw a tangent to this auxiliary circle from the center of the smaller circle as in Prob. 9. The required tangent will be parallel to this tangent, and the radii of contact will be perpendicular to it. For the second tangent draw a circle concentric as before with a radius equal to the sum of the radii of the given circles, and proceed in a similar manner as in the other case.)

Prob. 11. With center at $(1\frac{1}{4}'', 1\frac{1}{4}'')$ draw a circle through the point $(2'', 2'')$. Draw two circles, each of $\frac{3}{4}''$ radius, tangent to the given circle at the given point, one internal and the other external.

Prob. 12. With a center at $(1\frac{3}{8}'', 1\frac{3}{8}'')$ draw an arc A, passing through the point $(1\frac{3}{4}'', 2\frac{1}{4}'')$, and with a center at $(2\frac{3}{4}'', 1\frac{1}{2}'')$ and radius $= 1\frac{1}{8}''$ draw a circle B. It is required to draw a circle tangent to the given arc and the given circle at the given point on the arc. (Suggestion: Draw an arc concentric with the A arc having a radius equal to the difference between the radii of the given arc and circle respectively; join the center of the B circle and the point where the radius of the A arc through the given point meets the auxiliary arc; bisect this line. The center of the required circle will be at the intersection of these two lines.)

PLATE 7.

Prob. 13. Given three points ($\frac{1}{2}''$, $\frac{1}{2}''$), ($1\frac{3}{4}''$, $2\frac{3}{4}''$), and ($3''$, $1\frac{1}{4}''$). Draw a circumference tangent to the three sides of the triangle formed by joining the three points.

Prob. 14. Given a horizontal line $2\frac{1}{2}''$ long drawn from the point $x=\frac{1}{2}''$, $y=\frac{1}{2}''$ to the right.

Construct on this line an equilateral triangle and draw three circles tangent to the sides of the triangle and to each other.

Prob. 15. Inscribe an octagon within a given square whose side is $2\frac{1}{2}''$; the lower left-hand corner of the square is at ($\frac{1}{2}''$, $\frac{1}{2}''$). (Suggestion: With each corner as a center and a radius equal to half the diagonal draw arcs cutting the sides of the square).

Prob. 16. Inscribe a regular hexagon in a circle of $1\frac{1}{4}''$ radius whose center is at ($1\frac{3}{4}''$, $1\frac{3}{4}''$). (Suggestion: The radius of the circle equals the side of the hexagon.)

Prob. 17. Inscribe a regular pentagon in a circle of equal radius and like position as that in Prob. 16. Draw two diameters AB and DE, perpendicular to each other. Call C the center of the circle. Bisect AC in the point F; with F as a center and FD as a radius describe an arc cutting AB at G, then GD = the length of the side of the required pentagon.

Prob. 18. Draw a regular polygon of any number of sides (say seven) on a given base AB $1\frac{3}{16}''$ long and horizontal, whose middle point is ($1\frac{3}{4}''$, $\frac{9}{16}''$). With AB as a radius and A as a center describe a semicircle and divide it (in this case) into seven equal spaces; join A and the second point of division, 2, then this line will be a second side of the polygon. By Prob. 8 find the center of the circumscribing circle through the points A, B, and 2.

PLATE 8.

Prob. 19 Connect the following points by a continuous curve, the several parts of which are arcs of circles tangent to each other, the first arc being a semicircle: $(\frac{5}{16}'', 1\frac{1}{4}'')$; $1\frac{5}{16}''$, $\frac{1}{2}'')$; $(2\frac{1}{4}'', \frac{11}{16}'')$; $(2\frac{5}{16}'', 1\frac{1}{2}'')$; $(\frac{11}{16}'', 2\frac{1}{2}'')$; $(1\frac{13}{16}'', 2\frac{11}{16}'')$; $(2\frac{1}{2}'', 2\frac{11}{16}'')$; $(3'', 2\frac{1}{2}'')$; and $(3\frac{5}{16}'', 1\frac{3}{4}'')$.

Prob. 20. Draw an "egg oval" on a circumference whose diameter is $2''$ and whose center is $(1\frac{1}{2}'', 1\frac{3}{4}'')$. With AB as a radius and center at A describe the arc BE, and with DE as a radius and center at D describe the arc EE′.

Prob. 21. Draw a parabola on a horizontal axis $2\frac{1}{4}''$ long and a base $2''$ long; the intersection of the base and axis to be the point $(\frac{3}{4}'', 1\frac{3}{4}'')$. Divide AB and BD each into the same number of equal spaces and number the points of division as shown. Draw horizontal lines through the points of division on AB, and join A with the points of division on BD; where the lines correspondingly numbered intersect are points on the curve. These points are joined first by an irregular curve drawn free-hand in pencil and then in ink, using the scroll as a guide for the ruling-pen.

Prob. 22. Draw an ellipse whose major axis is $2\frac{1}{2}''$ long and whose minor axis is $1\frac{1}{2}''$ long; the intersection of the two axes to be the point $x = 1\frac{3}{4}''$, $y = 1\frac{3}{4}''$. Divide AB and AC each into the same number of equal spaces, join D′ with the points of division on AC, and D with the points of division on AB; where the lines correspondingly numbered meet are points on the curve.

Prob. 23. Draw an ellipse on axes the same as given in Prob. 22 by another method as indicated in the plate.

Prob. 24. Draw an hyperbola on a horizontal axis $1''$ long whose middle point is $(1\frac{3}{4}'', 1\frac{3}{4}'')$. Two methods of construction are shown, one on the left by a method similar to those used in finding points on the parabola and ellipse, and the other on the right as follows: Find F and F′ as shown, take any distance, as Aa, for a radius, and with F′ as a center describe an arc which passes through G, then with the difference between Aa and AA′ as a radius and F as a center describe an arc cutting the first arc in G. G is a point of the curve.

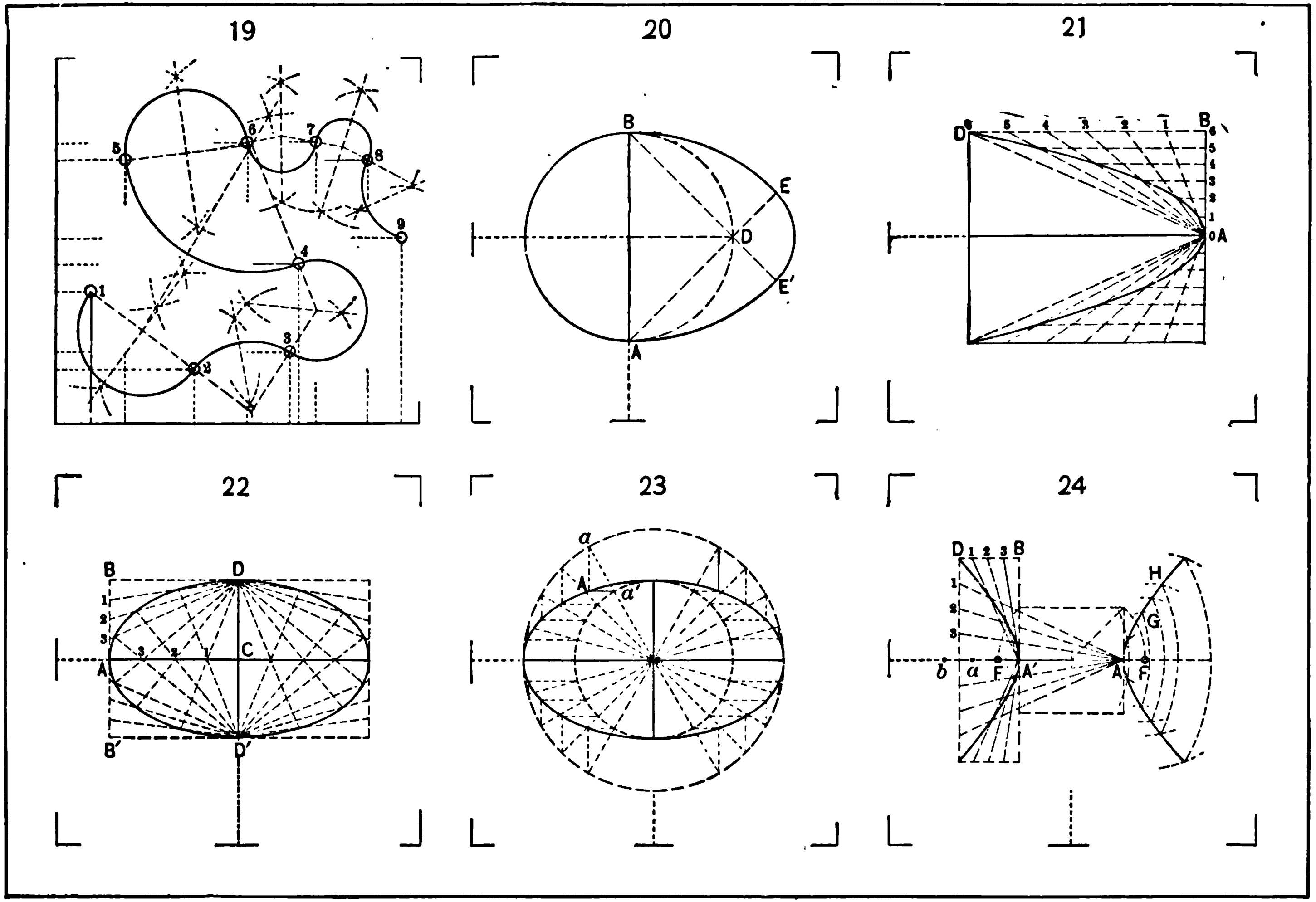

19
20
B
E
D
E′
A
21
D
B
0A
22
B
D
C
A
B′
D′
23
a
A
a′
24
D 1 2 3 B
H
G
b a F A′ A F

PLATE 9.

31. Construction and Uses of Scales.*

1. Construct scales of $\frac{1}{2}''=1'$, $\frac{3}{4}''=1'$ and $1\frac{1}{2}''=1'$. Draw three heavy horizontal lines $3''$ long, beginning at $x=\frac{1}{4}''$, and $1''$ apart, the first line to be at $y=2\frac{3}{8}''$; $\frac{3}{16}''$ above each heavy line draw a parallel, light line. Mark and figure the scales as shown in the accompanying plate.
2. Construct a diagonal scale of $\frac{1}{48}$, or $\frac{3}{4}''=1$ yd., to show yards, feet, and inches. Draw a rectangle $3''$ horizontal, $1''$ vertical, the left side at $x=\frac{1}{4}''$ and the bottom side at $y=\frac{1}{2}''$. Divide the left side into twelve equal spaces, and the bottom side into four equal spaces each $\frac{3}{4}''$ long, and the first of these spaces to the left into three equal spaces. Mark and figure as in the accompanying plate.
   (*a*) Lay off on a horizontal line drawn from ($\frac{1}{2}''$ $2\frac{5}{16}''$) 3 yds. 1 ft. 5 in. by this scale. Place one point of the spring dividers at the intersection of the vertical line through 3, and the fifth horizontal line from the bottom, and the other point at the intersection of the same horizontal and the diagonal through 1.
   (*b*) Lay off on another horizontal line drawn from $x=1''$, $y=2\frac{3}{4}''$, 1 yd. 2 ft. 10 in. All lines called "dimension-lines" which are drawn to show the limits of a measurement should be drawn light, full in red ink (shown in the plate as broken lines), the arrow-points at the extremities, and the numerals in black ink.
3. Construct a diagonal scale of inches, tenths, and hundredths of an inch. Draw a rectangle as in 2, mark and figure as in the accompanying plate.
   (*a*) Lay off on horizontal lines, situated as in 2, the distances $2.54''$ and
   (*b*) $1.46''$ by this scale. The principle is the same as in 2.
4. Construct a scale for reducing to $\frac{2}{5}$ and $\frac{3}{5}$. Draw a horizontal line $3''$ long from the point $x=\frac{1}{4}''$, $y=\frac{5}{8}''$; erect a perpendicular at the left end $2\frac{1}{2}''$ long and divide the perpendicular into five equal parts; join the third and fifth divisions with the right end of the horizontal line. Mark and figure as in the accompanying plate.
5. Construct a square whose side is $2\frac{1}{4}''$ and whose lower left-hand corner is at $x=\frac{5}{8}''$, $y=\frac{5}{8}''$. By the reduction scale of 4 draw two other squares inside the first, symmetrically placed, one whose sides are $\frac{3}{5}$ and the other whose sides are $\frac{4}{5}$ the sides of the first square. AB on the scale $=2\frac{1}{4}''$, AC $=\frac{3}{5}$ AB, and CB $=\frac{2}{5}$ AB.
6. Construct a Scale of Chords. Draw a horizontal line $3''$ long from the point $x=\frac{1}{4}''$, $y=\frac{5}{8}''$. At a point $x=1.12''$ (use the diagonal scale of 2) on this line as a center and with a radius $=2.13''$ draw a quarter of a circumference. Divide this arc into 9 equal parts and transfer the chords of these arcs to the horizontal line. All lines except those of the scale proper are drawn in red ink. Mark and figure as in the accompanying plate. (Suggestion: To measure a given angle with this scale, using a radius equal to the chord of 60° on the scale, describe an arc from the vertex of the angle as a center, measure the length of the chord on this arc between the sides of the angle and apply it to the scale. To lay off an angle from a given line at a point on the line, describe an arc from the point as a center and a radius equal to the chord of 60° on the scale, and on this arc lay off the chord of the required number of degrees as obtained from the scale.

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* The original drawings have been reduced in dimensions in reproducing Plates 9 and 10, so that the scales and distances in the plates are proportional only to the dimensions noted in the text.

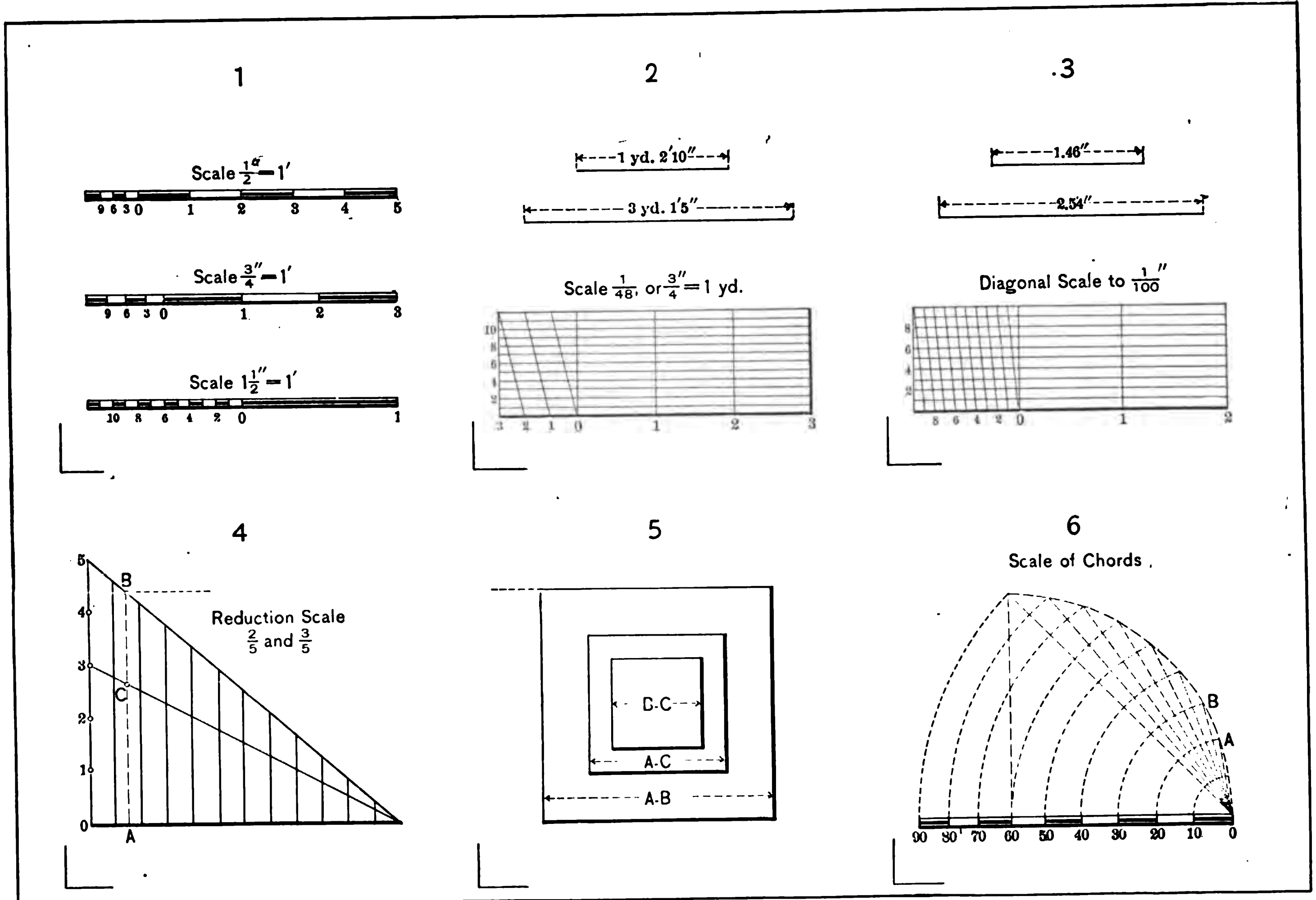
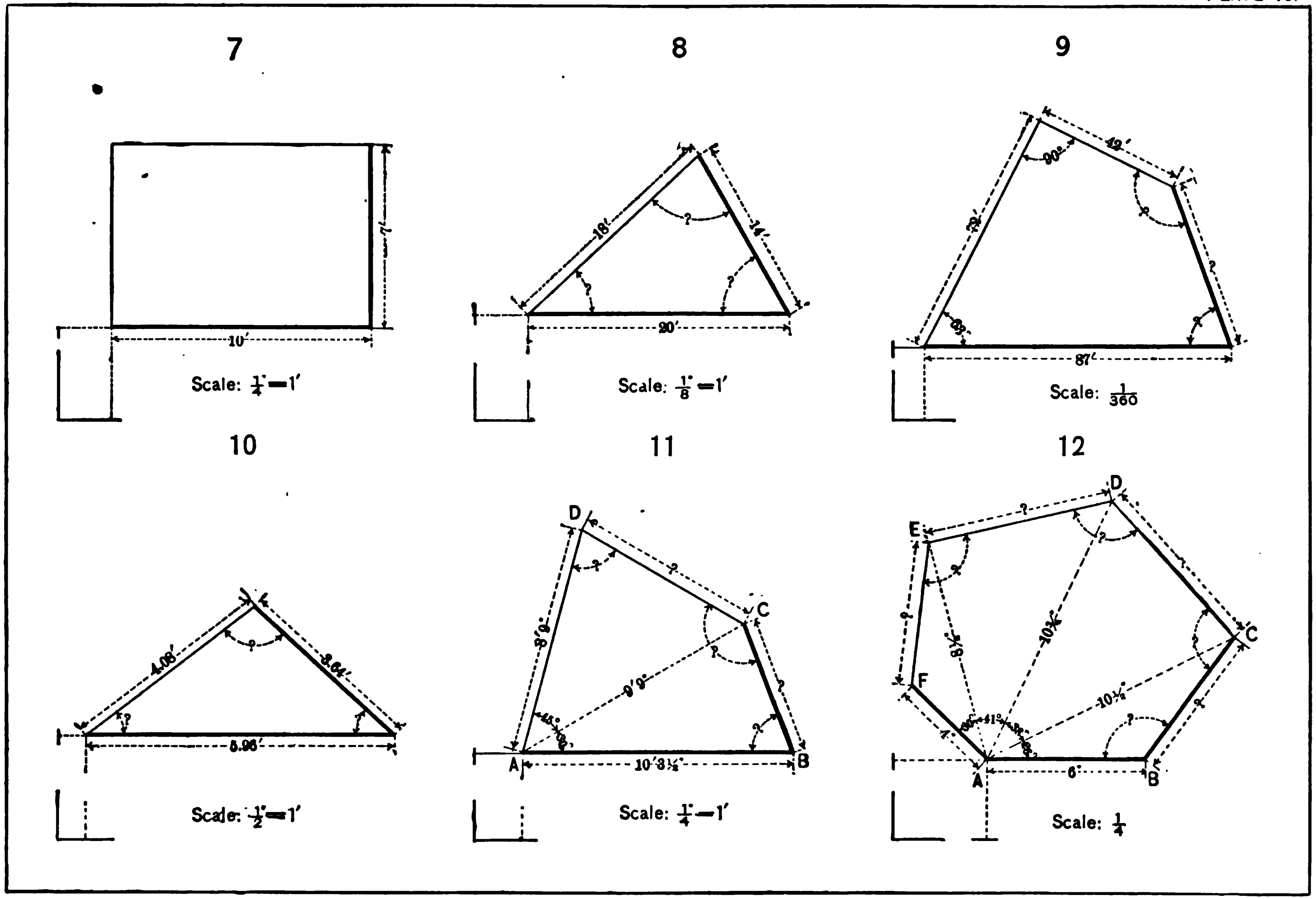

1
Scale $\frac{1}{2}'' = 1'$
9 6 3 0 1 2 3 4 5
Scale $\frac{3}{4}'' = 1'$
9 6 3 0 1 2 3
Scale $1\frac{1}{2}'' = 1'$
10 8 6 4 2 0 1
2
1 yd. 2′10″
3 yd. 1′5″
Scale $\frac{1}{48}$, or $\frac{3}{4}'' = 1$ yd.
3 2 1 0 1 2 3
3
1.46″
2.54″
Diagonal Scale to $\frac{1}{100}''$
8 6 4 2 0 1 2
4
Reduction Scale
$\frac{2}{5}$ and $\frac{3}{5}$
B
C
A
0 1 2 3 4 5
5
B-C
A-C
A-B
6
Scale of Chords
B
A
90 80 70 60 50 40 30 20 10 0

PLATE 10.

7. Scale $\frac{1}{4}''=1'$ (use the flat scale). With the longer side horizontal draw a rectangle $7'$ by $10'$.
   For the lower left corner $x=2'$, $y=3'\ 6''$.
8. Scale $\frac{1}{2}''=1'$. From the point $(4', 8')$ draw a horizontal line $20'$ long; upon this as a base construct a triangle whose other sides are $14'$ and $18'$. Measure the angles with the protractor and substitute the number of degrees for the question-mark.
9. Scale $\frac{1}{360}$ or $1''=30'$ (use the triangular scale of 30 parts). Draw a polygon; 1st, from the point $(9', 20')$ draw a horizontal line $87'$ long; 2d, from the same point a line $72'$; 3d (continuing around to the right), a line $42'$; the fourth side to close the polygon. The angle between the first and second sides $=63°$; between the second and third sides $=90°$.
   Determine with the scale the fourth side, and with the protractor the two adjacent angles, and insert them in their proper places.
10. Scale $\frac{1}{2}''=1'$ (use the diagonal scale in Plate 9). From the point $x=0.62'$, $y=2'$ draw a horizontal line $5.96'$ long; upon this as a base construct a triangle whose other sides are $4.08'$ and $3.64'$ respectively.
    Measure the angles with the scale of chords in Plate 9.
11. Scale $\frac{1}{4}''=1'$. Construct a polygon.
    For the point A, $x=1'\ 10''$, $y=3'\ 4''$.
    Take AB on a horizontal line to the right, $10'\ 3\frac{1}{2}''$.
    Take AC $=9'\ 9''$ on a line making an angle of $30°$ with AB (use the scale of chords).
    Take AD $=8'\ 9''$ on a line making an angle of $45°$ with AC.
    Measure the sides DC and CB with the scale and the angles ADC, DCB, and ABC with the scale of chords.
12. Scale $\frac{1}{4}$ (use a reduction-scale made on a separate piece of paper). Construct a polygon.
    For the point A, $x=3\frac{1}{2}''$, $y=3''$.
    Take AB on a horizontal line to the right, $6''$.
    Take AC $=10\frac{1}{2}''$ on a line making an angle of $26°$ with AB (use the protractor).
    Take AD $=10\frac{3}{4}''$ on a line making an angle of $38°$ with AC.
    Take AE $=8\frac{1}{2}''$ on a line making an angle of $41°$ with AD.
    Take AF $=4''$ on a line making an angle of $30°$ with AE.
    Measure the sides BC, CD, DE, and EF with the same scale; also the angles ABC, BCD, CDE, DEF, and EFA with the protractor.


7
10'
7'
Scale: 1/4" = 1'
8
18'
14'
20'
Scale: 1/8" = 1'
9
42'
90°
72'
87'
Scale: 1/360
10
4.08'
3.64'
5.96'
Scale: 1/2" = 1'
11
D
C
A
B
8'0"
9'9"
45°
10'3½"
Scale: 1/4" = 1'
12
D
E
C
F
A
B
10½"
8½"
6"
41°
Scale: 1/4

CHAPTER IV.

PROJECTIONS.

32. Definitions.—In representing an object by drawings on a plane surface, *if the projecting lines of the several points are perpendicular to the plane of projection the projection is called orthographic,* and such drawings are used as *working drawings.* This chapter treats of orthographic projections only.

The projection of an object may be made on one plane only, or on several planes in order to show its true form; generally at least two projections are made, one on a *horizontal* and the other on a *vertical* plane. These planes are called the *principal planes of projection.* A plane of projection which is perpendicular to both the horizontal and vertical planes is called a *perpendicular* or *profile plane,* and a plane of projection having any other position with reference to the horizontal and vertical planes is known as a *supplementary plane.*

It is convenient to consider that the object may be placed in any desired position with relation to the horizontal plane of projection; the vertical plane of projection may then be taken perpendicular to the horizontal plane in any possible relation to the object. The profile plane is determined in its position as perpendicular to both the horizontal and vertical planes.

The plane of the paper is assumed to coincide with each plane of projection in turn, as the drawing is made on that particular plane.

33. Conventions.—In these drawings each object to be represented by its projections will be assumed in the *third angle,* i.e., below the horizontal and behind the vertical plane. The *horizontal projection* (top view or plan) *will be shown above the vertical projection* (front view or elevation).

H will be used to designate the *horizontal plane of projection,* V the *vertical plane of projection,* and P the *profile,* or *perpendicular, plane of projection.* When a *supplemental plane of projection* is used, it will be referred to as S.

The lower and left border-lines of the plate are respectively *reference-axes* for locating points on the drawing (Art. 19).

Any point referred to on the object will be designated by a *capital letter, its projection on the several planes* by

the *same capital letter with a subscript letter designating which plane,* thus: A_h represents its projection on H; A_v, its projection on V; A_p and A_s, its projections on P and S respectively. For the same point in a development (pattern) the same capital letter will be used without a subscript.

Whenever a line in a drawing represents an edge of a face which is in view (the planes of projection are considered transparent), it is to be drawn *full in black.* If it is not in view, it is invariably to be made a *light dash-line in black.* All construction-lines when shown in the finished drawing must be *light, full lines in red.* Lines connecting the different projections of the same point must be *light dot-lines in red.* All dimension-lines should be drawn as stated in Art. 31, 2 (*b*).

34. **Shade-lines.**—When the object to be represented is made up of plane surfaces or faces, a distinction is made in the intensity of the lines *representing edges that are in view. Whenever a line represents an edge joining two faces both in the light or both in the shade, it is drawn light. Whenever it joins two faces, one in the light and the other in the shade, it is made heavier.* These heavier lines are called *shade-lines* (sometimes called shadow-lines).

To determine accurately when a face is in the shade or in the light involves principles of Descriptive Geometry with which, for this course, it is not assumed the student is familiar. However, an example or two may make plain the method of determining shade-lines. It is conventional to regard the rays of light as parallel, and coming over the left shoulder in the direction which the diagonal of a cube has when the faces of the cube are parallel to the H and V planes. The *projections of rays on H* would then have a direction making *an angle of 45 degrees, with a horizontal line* inclining upwards to the right, and the *projections on V an angle of 45 degrees* inclining downwards to the right.

In some drawings it is easy to determine by inspection which faces are in the light and which are in the shade. For example, in Fig. 7 the hexagonal prism has its bases parallel to H, and V is taken parallel to a face. R_h and R_v show the direction of the projections of the rays of light on H and V respectively.

Draw the projection on H of a ray of light, as Q_h, moving in the direction of the arrow; then any line, as Q_v, drawn parallel to R_v may be taken as the projection of the same ray on V. This ray of light, Q, pierces the face of the prism, whose projection on H is the line A_h B_h, and its projection on V the rectangle A_v B_v A_v' B_v', in the point whose projection on H is G_h, and whose projection on V is G_v. If produced it would afterwards pierce the face B_h C_h–B_v C_v B_v' C_v' in the point K_h–K_v, showing that this face is in the shade while the first-mentioned face is in the light.

Again, the ray S_h–S_v pierces the back face F_h E_h–F_v A_v F_v' A_v' before piercing any other face, showing that the face is in the light.

In a similar manner it may be determined which of the other faces are in the light and which are in the shade.

When the same prism is tipped as in Fig. 8 to find whether the face A B E F is in the light or in the shade by the intervention of the face A′ B′ E F between it and the source of light, proceed as follows: Draw the projection on H of a ray of light, as the line R_h, making an angle of 45 degrees with a horizontal line, and crossing the H projections of both faces. From the point H_h, where this line crosses the line A_h B_h, let fall a perpendicular meeting A_v' B_v' at H_v; similarly find M_v and N_v. Join N_v with H_v and M_v respectively. The line N H lies in the face A B E F, and M N in the face A′ B′ E F. Now, if from any point on M_v N_v, as G_v, a line R_v be drawn making an angle of 45 degrees with a horizontal line, inclining downwards to the right, it will be the projection on V of a ray of light, which is one of the many rays of which R_h is the common projection on H. If R_v, when prolonged in the direction of the arrow, meets the line N_v H_v, prolonged if necessary, it shows that this ray of light pierces the face A′ B′ E F before it does the face A B E F; which means, therefore, that the face A B E F is in the shade. If, however, R_v when drawn in the opposite direction had met the line N_v H_v, it would have shown the face A B E F to be in the light.

Again, to find whether the face A′ B′ C′ D′ prevents the light from shining on the face D′ C′ K L, use the same horizontal projection of a ray of light, R_h. Where R_h meets the line A_h' B_h' at M_h, let fall a perpendicular meeting A_v' B_v' at M_v; and where it meets the line C_h' D_h' at M_h let fall a perpendicular meeting the line C_v' D_v' at P_v; also where it meets K_h L_h at N_h let fall a perpendicular meeting K_v L_v at Q_v. Join P_v with M_v and Q_v respectively. At any point on M_v P_v, as W_v, draw R_v' parallel to R_v. It does not intersect P_v Q_v; therefore the face D′ C′ K L is in the light.

By inspection it is clear that the end face, or base, to the left, and also the face A′ B′ C′ D′, are in the light. Also it is clear that the other base and the face A B C D are in the shade.

In the examples given it was shown that the face A B E F is in the shade, and that the face D′ C′ K L is in the light. Similarly find that the remaining face C D L K is in the shade by the intervention of the face C′ D′ L K.

In the drawings to follow, the shade-lines as determined in the projections on H and V will not change in the projections on the P and S planes.

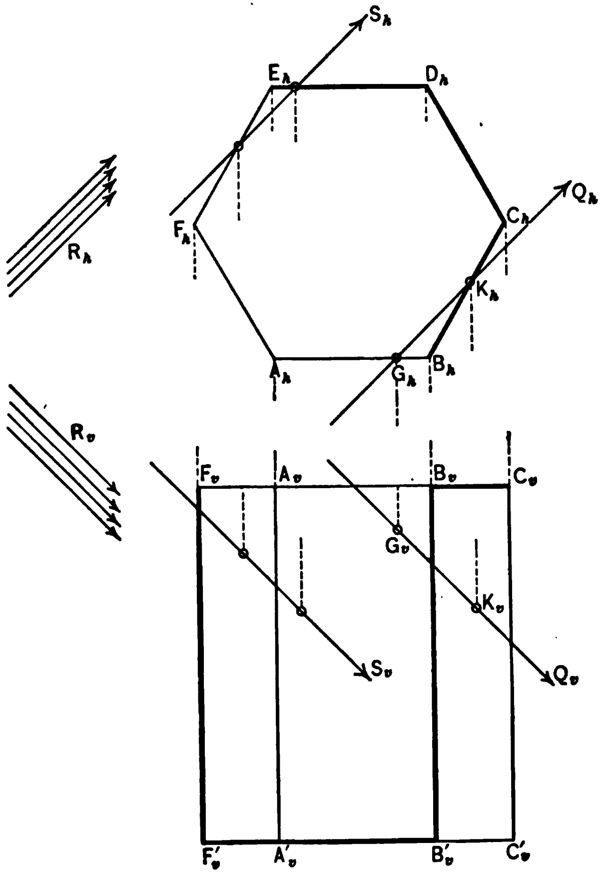

S_h
E_h
D_h
Q_h
R_h
F_h
C_h
K_h
A_h
G_h
B_h
R_v
F_v
A_v
B_v
C_v
G_v
K_v
S_v
Q_v
F'_v
A'_v
B'_v
C'_v

FIG. 7.

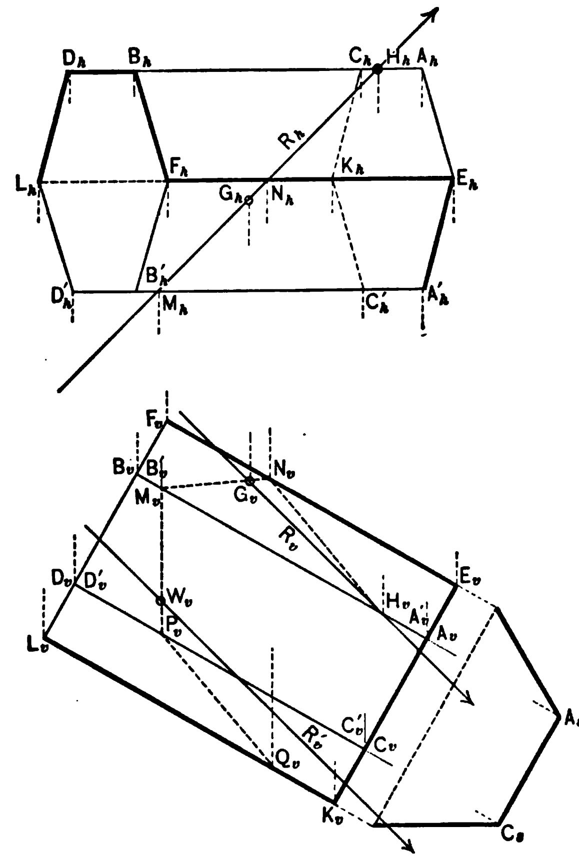

D_h
B_h
C_h
H_h
A_h
R_h
L_h
F_h
K_h
E_h
G_h
N_h
D'_h
B'_h
M_h
C'_h
A'_h
F_v
B_v
B'_v
N_v
M_v
G_v
R_v
E_v
D_v
D'_v
W_v
H_v
A'_v
A_v
L_v
P_v
A_s
C'_v
R'_v
C_v
Q_v
K_v
C_s

FIG. 8

PLATE 11.

35. **The Cube.**—Draw the projections on H and V of a cube whose edge is 2″, in each of the following three positions:

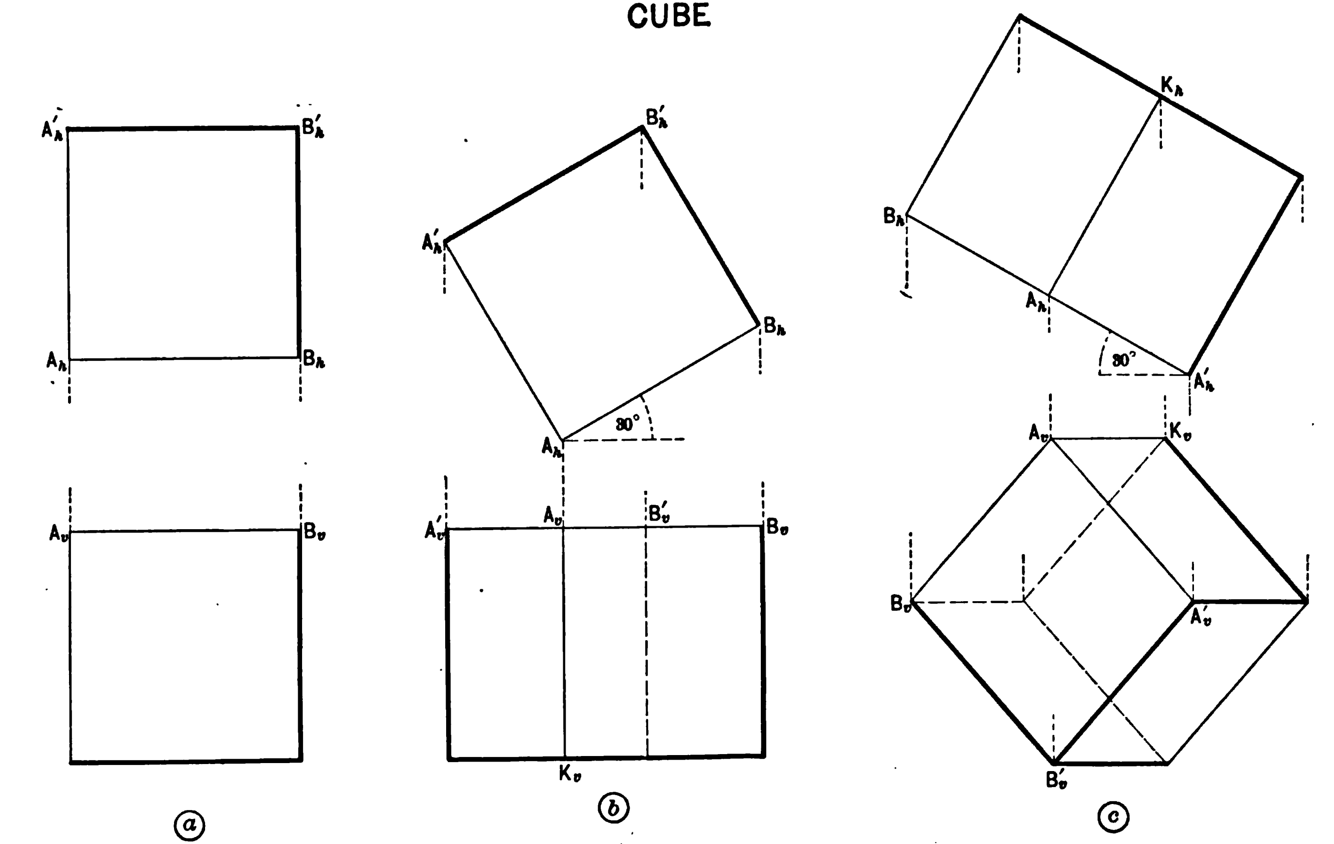
(*a*) When one face of the cube is parallel to H, and V is parallel to an adjacent face.
In the projection on H, the nearer left corner of the top face, A_h, is at (1″, 5″).
In the projection on V, A_v is at (1″, 3.5″).

(*b*) When one face of the cube is parallel to H, and V makes an angle of 30 degrees with an adjacent face.
In the projection on H, the nearest corner, A_h, is at (5.25″, 4.25″).
In the projection on V, A_v is at (5.25″, 3.5″).

(*c*) When the top edge, A K, of the cube is parallel to H, the adjacent faces each making an angle of 45 degrees with H; and V makes an angle of 60 degrees with the edge A K. Draw the projection on H first.
In the projection on H, A_h is at (9.5″, 5.5″).
In the projection on V, A_v is at (9.5″, 4.25″).
The front face is perpendicular to H, therefore its projection on H is a straight line A_h' B_h', equal to the diagonal of a face. In the projection of this face on V the distance A_v B_v' is equal to the diagonal of a face.
The edge A K is parallel to H, therefore its projection on V is the horizontal line A_v K_v, and its length is equal to the perpendicular distance between the projecting lines A_v A_h and K_v K_h.
All edges that are parallel to each other are projected in lines that are parallel on both H and V.

36. **Sections.**—If an object or body be intersected or cut by a plane, and the part of the body on either side of the cutting plane be removed, the surface thus exposed is called a *section*. The plane is called a *section plane*, and its intersection with a plane of projection is called its *trace* on that plane and is indicated by a *black dash-and-two-dot line*.

CUBE



A'h
B'h
Ah
Bh
Av
Bv
(a)
A'h
B'h
Bh
Ah
30°
A'v
Av
B'v
Bv
Kv
(b)
Kh
Bh
Ah
30°
A'h
Av
Kv
Bv
A'v
B'v
(c)

PLATE 12.

In this plate copy in pencil in the same positions the projections of the cube as they are on the preceding plate, and show sections as follows:

(*a*) In this position pass a section plane perpendicular to V and making with H an angle of 60 degrees.

In the projection on V, cut the edge A_v B_v, $\frac{3}{4}''$ to the right of A_v, and show the section projected on H.

The wedge projected on V in A_v D_v C_v is removed and the section is shown on H in C_h G_h D_h K_h.

(*b*) In this position pass a section plane perpendicular to H and parallel to V.

In the projection on H, let E_h F_h be 4.75″ above the horizontal axis of the plate. Show the section projected on V. The wedge projected on H as A_h E_h F_h is removed and the section is shown on V in E_v L_v F_v N_v.

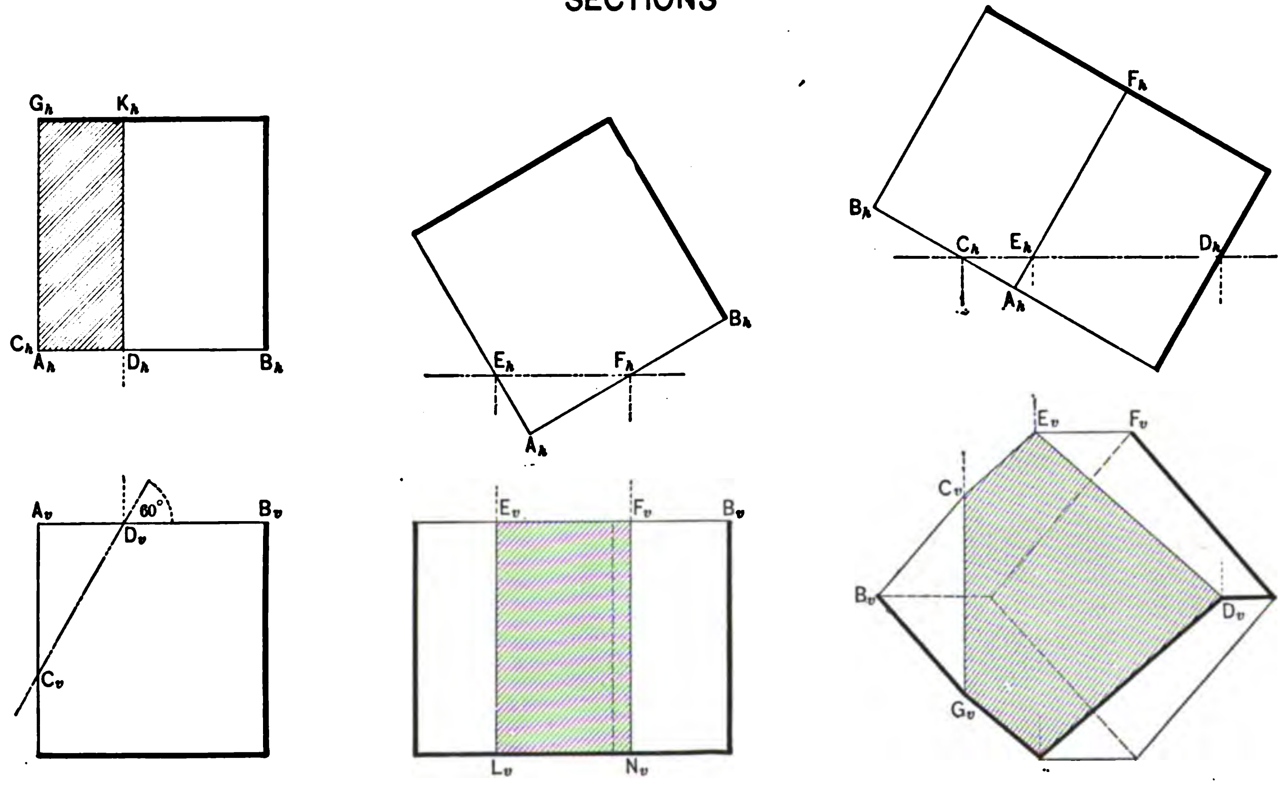
(*c*) In this position pass a section plane perpendicular to H and parallel to V.

In the projection on H, let C_h D_h be 5.75″ above the horizontal axis of the plate. Show the section projected on V.

In the projection on H, all that portion of the cube in front of the plane C_h D_h is removed. The edge whose projection on H is A_h B_h is cut at C, which is shown at C_v in the projection on V. The front face of the cube is cut in a vertical line; its projection on V is C_v G_v. The edge whose projection on H is A_h F_h is cut at E, shown as E_v in the projection on V. Similarly D_v is found. The boundary of the section is obtained by joining consecutively the points so determined.

Notes: In the projection on which the section is shown, ink in no lines representing parts removed. "Section-lines" are parallel lines drawn on a section, about $\frac{1}{16}''$ apart, making some angle with a horizontal line, usually 45 degrees. The trace of a section plane should be a dash-and-two-dot line drawn, in black.

SECTIONS



(a) (b) (c)

PLATE 13.

Sections (continued).—Draw the projections on H and V of a cube whose edge is 2″, and which has a square hole cut through it between two opposite faces. The side of the square hole is 1″.

(*a*) One face of the cube is parallel to H, and V makes with an adjacent face an angle of 15 degrees. The edges of the hole running through the cube are parallel to V.

In the projection on H, the nearer, left corner of the top face, A_h, is at (1.5″, 5″).

In the projection on V, A_v is at (1.5″, 3.5″).

Draw the projection on H first.

(*b*) Two adjacent faces each make an angle of 45 degrees with H, and V is parallel to a face. The edges of the hole which run through the cube are perpendicular to V.

In the projection on H, the highest, front corner, A_h, is at (6.5″, 5″).

In the projection on V, A_v (not shown but coinciding with E_v') is at (6.5″, 4.3″).

Cut this position of the cube by a section plane perpendicular to H and making an angle with V of 30 degrees, inclined backwards to the right, and cutting the front face one-half inch to the right of the corner B.

Show the section on the projection on V.

Draw the projection on V first.

(*c*) Draw a projection on the profile plane to the right of this second position of the cube, and show the section.

In the projection on P, A_p is at (9″, 4.3″).

$H_p\ E_p' = H_h\ E_h$; $G_p\ K_p = G_h\ K_h$; $F_p\ K_p = F_h\ L_h$; $D_p\ K_p = D_h\ M_h$; $E_p'\ E_p = E_v'\ E_v$; $C_p\ B_p =$ the perpendicular distance between the projecting lines of C and B on P, shown by the perpendicular distance between $C_v\ C_p$ and $B_v\ B_p$. Similarly the lengths of other lines in the projection on P may be found.

CUBE WITH HOLE

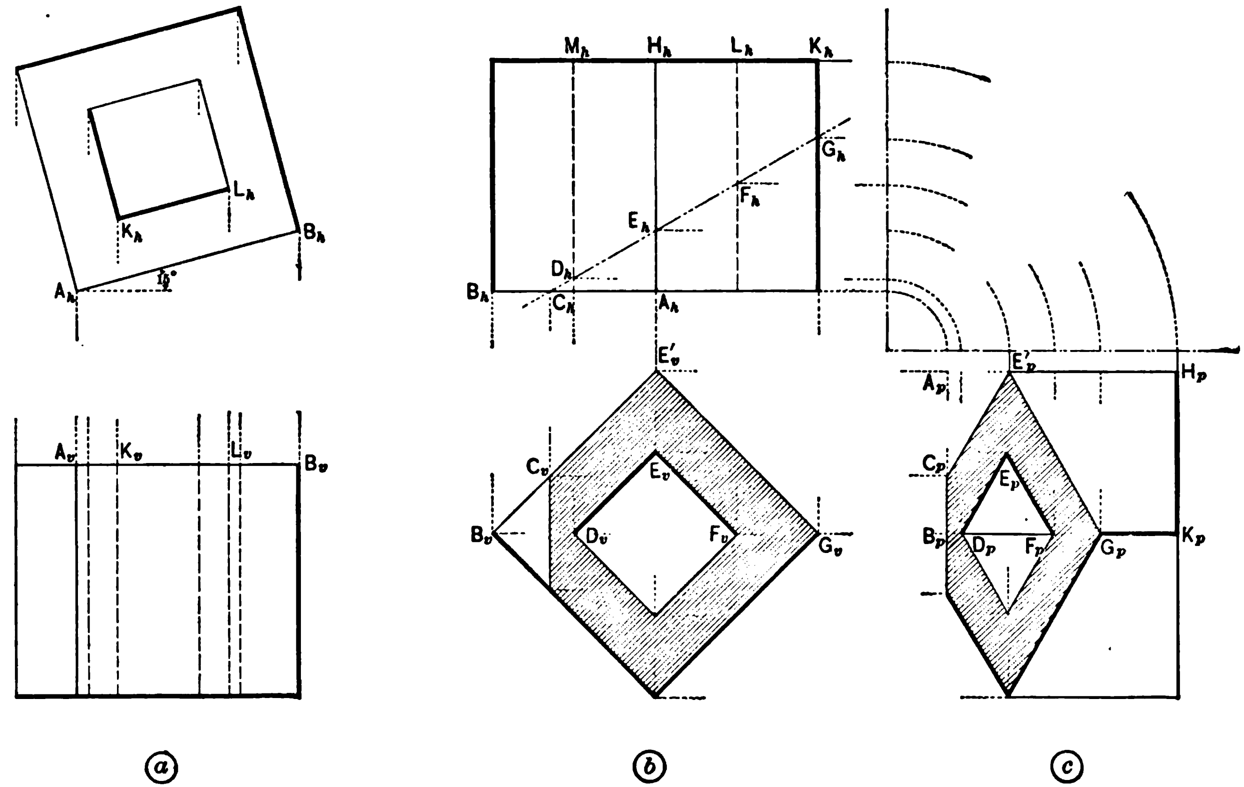


PLATE 14.

37. **Rectangular Prism.**—Draw the projections on H and V of a rectangular prism whose base is a rectangle 2″ by 1.5″ and whose height is 3″.

(*a*) When the base is parallel to H, and V is parallel to the faces which are 2″ wide.

In the projection on H, the front, left corner of the upper base, A_h, is at (1″, 5.5″).

In the projection on V, A_v is at (1″, 4.5″).

(*b*) When the narrowest faces make with H an angle of 30 degrees (inclining upwards to the right), and V is parallel to the broadest faces. Draw the projection on V first.

In the projection on H the lowest, front corner, C_h, is at (5.7″, 5.5″).

In the projection on V, C_v is at (5.7″, 1.5″).

Cut this position of the prism by a section plane perpendicular to H and making an angle with V of 30 degrees, extending backwards to the right. F_h is ½″ from B_h.

Show the section in the projection on V.

(*c*) Draw the projection of this second position on the P plane to the right, and show the section.

In the projection on P, C_p is at (10″, 1.5″).

$A_p E_p = A_h E_h$; $B_p F_p = B_h F_h$; $D_p G_p$ = the perpendicular distance between the projecting lines on P of D and G, shown by the perpendicular distance between $D_v D_p$ and $G_v G_p$.

RECTANGULAR PRISM

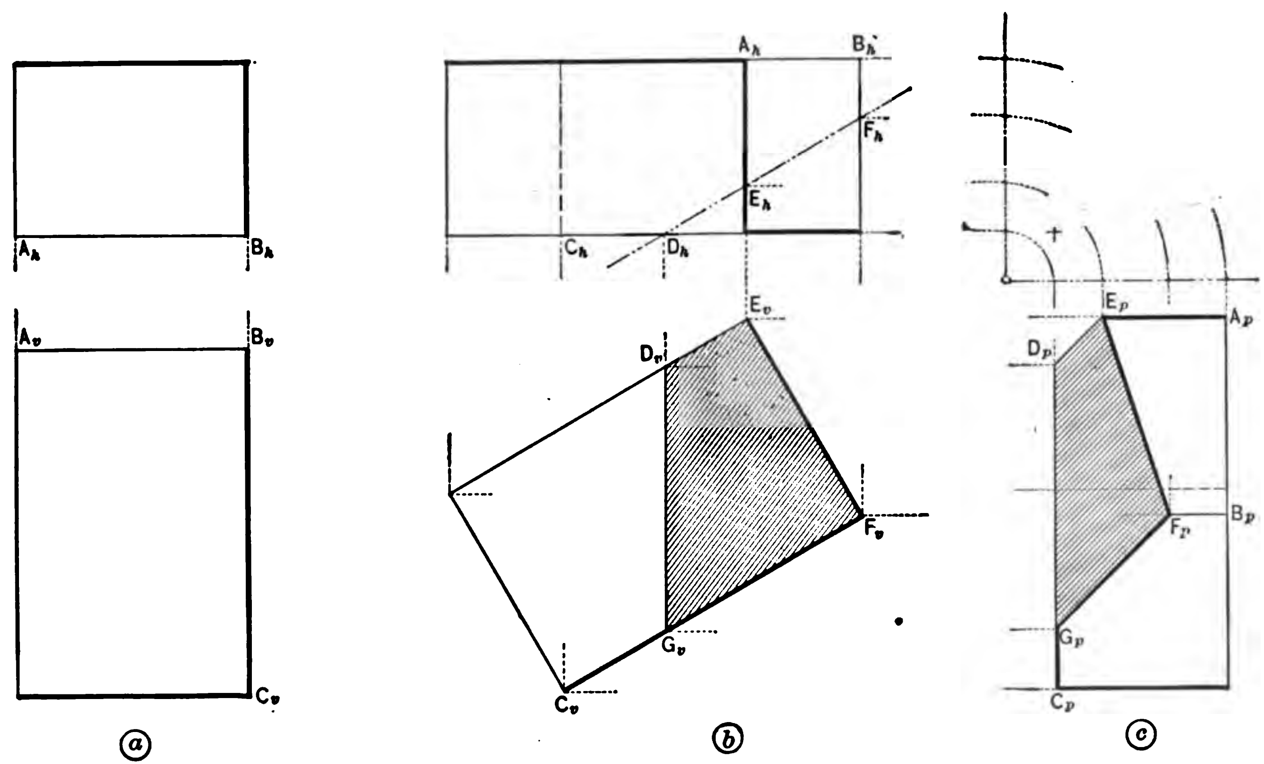


PLATE 15.

38. **Supplemental Projections.**—Draw the projections on H and V of the same prism that is given in Plate 14.

(*a*) When the base is parallel to H, and V makes an angle of 30 degrees with the widest face.

In the projection on H, the nearest corner, A_h, of the upper base is at (7″, 5″). Draw the projection on H first.

In the projection on V, A_v is at (7″, 4.5″).

(*b*) Draw to the right a profile of this prism.

In the projection on P, A_p is at (9″, 4.5″).

Assume in (*a*) a section plane perpendicular to V, and making an angle of 30 degrees with H. Let the intersection of this plane with V (called the V trace) cut the edge farthest to the right at 1.1″ down from the top. Show the section only on the P projection in (*b*).

(*c*) Draw a supplemental projection of the prism to the left on a plane perpendicular to V, and making an angle of 60 degrees with H.

In the projection on S, A_s is at (4.5″, 5.9″).

In the projection on S, the projected length of the edge A K on S is found by projecting the line $A_v K_v$ on a line through A_s, making an angle of 60 degrees with a horizontal line. The point *a* is the intersection of a horizontal through A_h and the 60° line through A_s. The projections of the other parallel edges are equal and parallel to $A_s K_s$. The perpendicular distance, *a b*, between the edges projected in $A_s K_s$ and $B_s L_s$ is found by projecting the line $A_h B_h$ on a line perpendicular to a horizontal, as *a b*. Similarly the perpendicular between the lines $B_s L_s$ and $C_s N_s = c\, b = c_1\, b_1$. The projections of opposite sides of the prism are equal parallelograms.

SUPPLEMENTARY PROJECTION

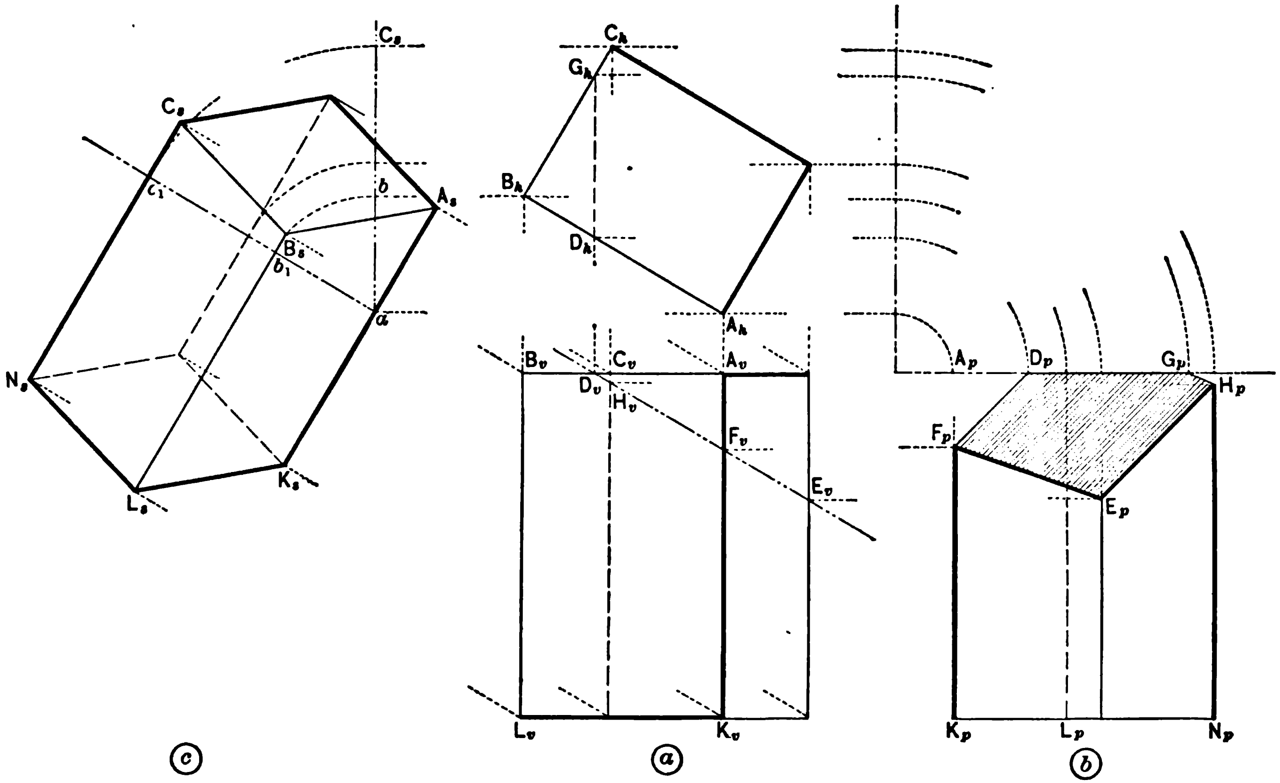


PLATE 16.

39. **Hexagonal Prism.**—Draw the projections on H and V of a prism 3″ long, whose base is a regular hexagon with a side of 1″.

(*a*) When the base is parallel to H, and V is parallel to a face.

In the projection on H, the upper corner of the front face, A_h, is at (3″, 5″). Draw the projection on H first.

In the projection on V, A_v is at (3″, 4.5″).

(*b*) Draw a projection of this position of the prism on the P plane to the right.

In the projection on P, A_p is at (4.25″, 4.5″).

The perpendicular distance between the edges of the faces in front of the P plane, as $A_p B_p$, $= a\ b$.

(*c*) Draw the projections on H and V of the same prism when the edges make an angle of 30 degrees with H, and V is parallel to one face.

In the projection on H the corner A_h is at (7.75″, 5″).

In the projection on V, A_v is at (7.75″, 4″).

Draw the projection on V first. (It is the same figure as the projection on V in (*a*).)

(*d*) Draw the projection of one-half of the lower base on a supplemental plane parallel to and ½″ from it, as $D_s F_s$. This supplementary projection may be used to get the projection on H, for $E_s G_s = E_h G_h$.

HEXAGONAL PRISM

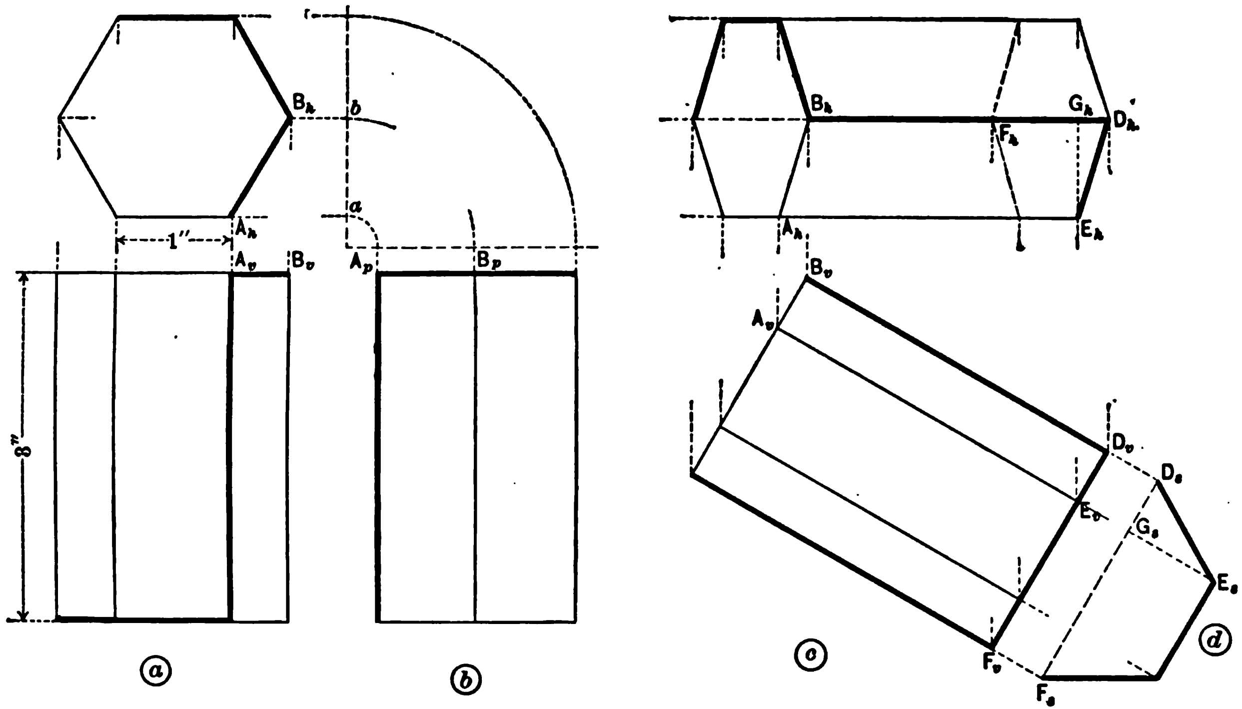


PLATE 17.

40. Development.—Draw again the projections (*c*) and (*d*) of the previous plate, cut the prism by a section plane, and develop a portion of the prism.

(*a*) The V trace, $A_v D_v$, of the section plane is parallel to H, being 3.5″ from the horizontal axis of the plate. Show the section on the horizontal projection.

In the projection on H, F_h is at (2.25″, 5″).

In the projection on V, F_v is at (2.25″, 4″).

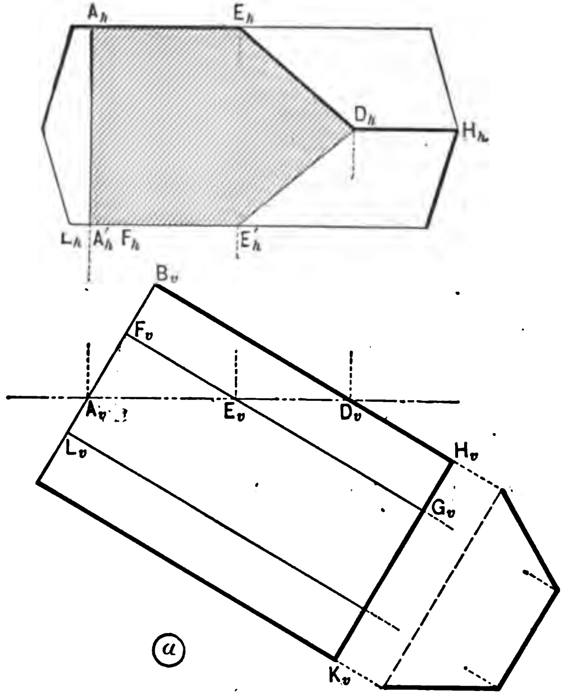
(*b*) Develop, or make a pattern of, the side faces of the larger part of this prism, that part below the section (omitting the base and the section).

Suppose the prism to be hollow. Since all of its faces are plane surfaces, they may be made to coincide with a common plane by causing each face in turn to come into that plane by rotation on the uniting edges in succession.

Let K be at (11″, 4.25″). The sides of the base will fall in the line H H. The edges of the side faces will be perpendicular to the base, H H, and be separated by a distance equal to the side of the hexagon of the base. The true lengths of the edges will be found in their projections on V, because they are parallel to V. That is, D H = $D_v H_v$, etc., and A′ L = $A_v L_v$ for the same reason.

If this pattern were cut out of the paper following its outline and bent on the lines of the uniting edges, it would take the form of the lower part of the prism. It could thus be used as a pattern to mark out on sheet metal or other thin material the surface of the prism.

DEVELOPMENT



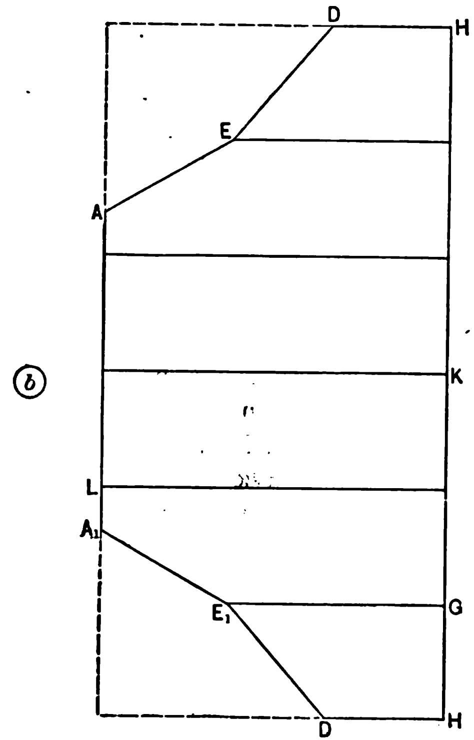


PLATE 18.

41. **Intersections.**—In the intersection of surfaces it is required to show the projection of lines which are common to both surfaces, that is, their intersection.

Draw the projections of two prisms intersecting each other, one with a hexagonal base and the other with a square base, with dimensions as shown on the plate.

(*a*) The base of the hexagonal prism is parallel to H, and V is parallel to a face. Draw the projection on H first.

In the projection on H, the center of the top base is at (2″, 6″).

In the projection on V, the center of the top base is at (2″, 4″).

The other prism is behind the first; its axis intersects and is perpendicular to the first prism and is perpendicular to V.

The faces make an angle of 45 degrees each with H.

Draw the projection on V first.

In the projection on H, the middle of the base is at (2″, 7.75″).

In the projection on V, the middle of the base is at (2″, 2.75″).

(*b*) Draw a projection on a P plane to the right.

In the projection on P, the middle of the upper base is at (5″, 4″).

$C_p c'' = C_h c$ or $C_p L_p = C_h L_h$. The distance from B_p to the axial line $C_p c''$ is equal to the perpendicular distance between the projecting lines from B_v and C_v on $D_p E_p$.

(*c*) Develop the side faces of the hexagonal prism and trace on the pattern the intersecting lines of the two prisms.

The corner B′ of the upper base is at (11.5″, 4.25″).

$B' B = B_v' B_v$; $B' G = B_v' G_v$; $B' C' = B_h C_h$.

INTERSECTION

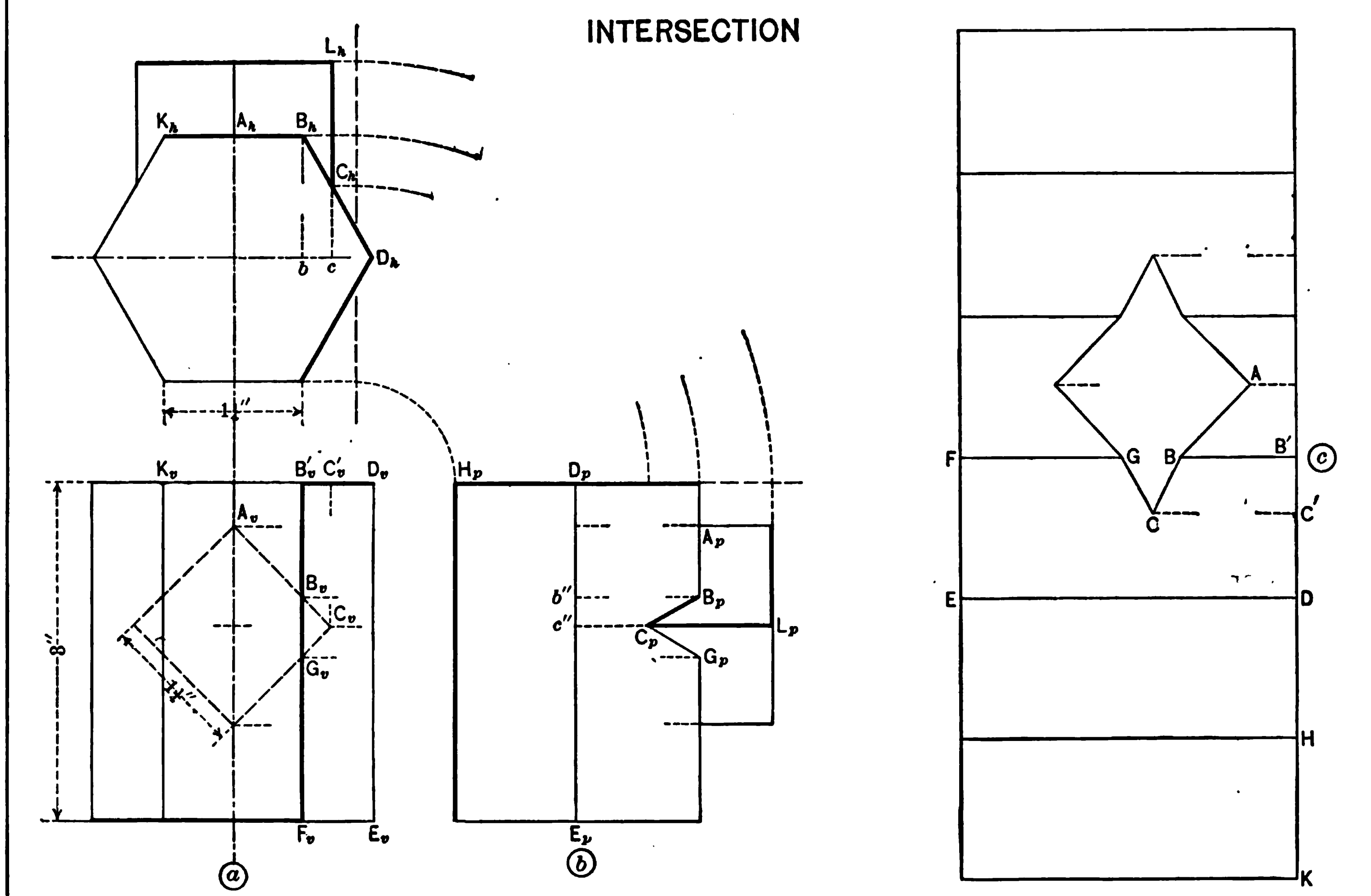


PLATE 19.

42. Oblique Intersections. — Draw the projections of two prisms intersecting each other obliquely. The base of each is a regular hexagon. The dimensions are shown in the plate.

(*a*) In the upright prism the base is parallel to H, and V is parallel to a face.

In the projection on H, the center of the upper base is at (6″, 6″).

In the projection on V, the center of the upper base is at (6″, 4.25″).

Draw the projection on H first of the upright prism.

The axis of the oblique prism inclines to H at an angle of 45 degrees downwards to the right and is parallel to V. The plane of its base is perpendicular to its edges.

In the projection on H, the middle of the base, C_k, is at ($4\frac{1}{8}$″, 6″).

Draw the projection of one-half of the base of the oblique prism on a supplemental plane, parallel to the base and $\frac{1}{4}$″ from it.

In the projection on S, the center of the base, C_s is at (4″, 3.75″).

Draw the projection on the S plane first, on the H plane second, and finally on the V plane.

(*b*) Draw the projections of these prisms on a P plane to the left. The axial line, *a b*, is 2″ from the vertical axis of the plate. Study the figures on the plate to determine the projected distances between edges and their ends.

(*c*) Develop the oblique prism and trace on the pattern the lines of intersection with the upright prism. The line of the top base is 11″ from the vertical axis of the plate, and K L is at $y=4.25$″. The true lengths of the edges are found in the projections on V.

OBLIQUE INTERSECTION

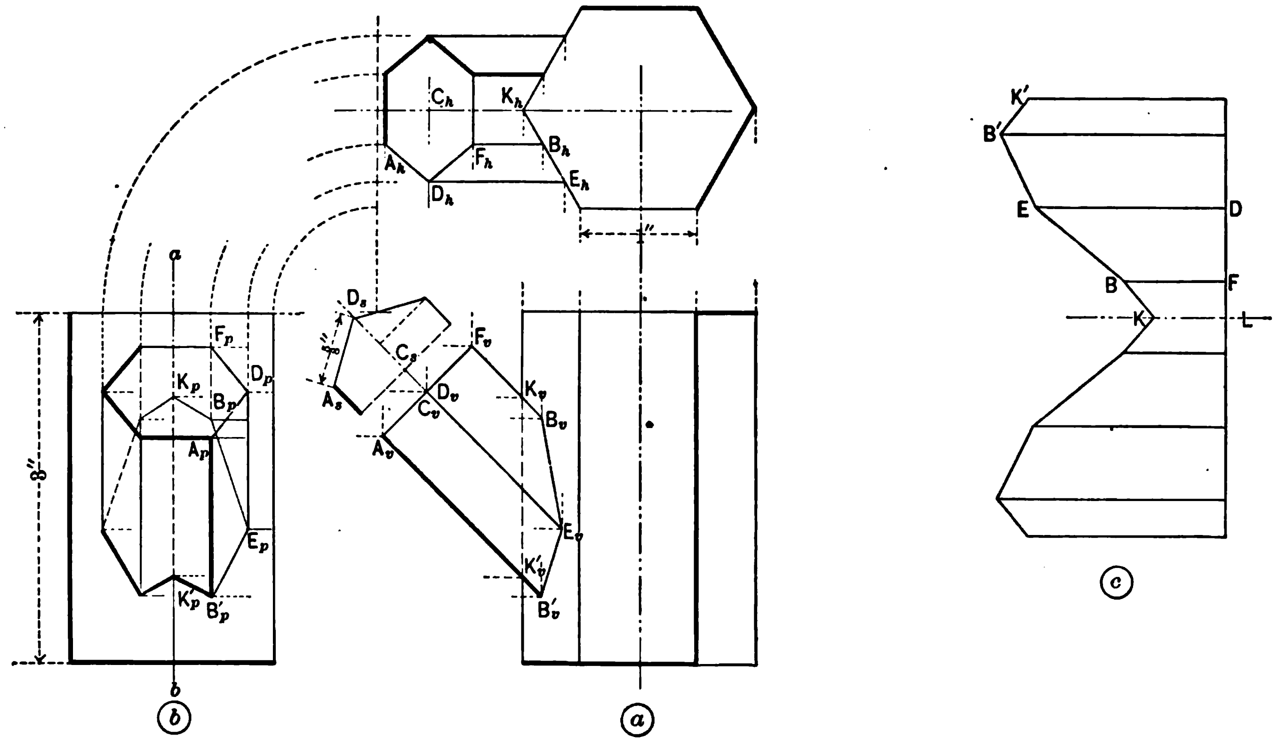


PLATE 20.

43. **The Pyramid.**—Draw the projections on H and V of a pyramid in three different positions, with dimensions as shown in (*a*).

(*a*) When the base is parallel to H, and V is parallel to a side of the base.

In the projection on H, V_h is at (2″, 6″).

In the projection on V, V_v is at (2″, 4.5″).

(*b*) When the base is parallel to H, and V makes an angle of 30 degrees with a side of the base. Draw the projection on H first.

In the projection on H, V^h is at (6″, 6″).

In the projection on V, V_v is at (6″ ,4.5″).

(*c*) When the plane of the base makes an angle of 30 degrees with H, and V is parallel to a side of the base and also parallel to the axis. Draw the projection on V first.

In the projection on H, V_h is at (11.5″, 6″).

In the projection on V, V_v is at (11.5″, 4.25″).

$B_h C_h$ = the side of the base, because the side is parallel to H and is, therefore, projected in its true length.

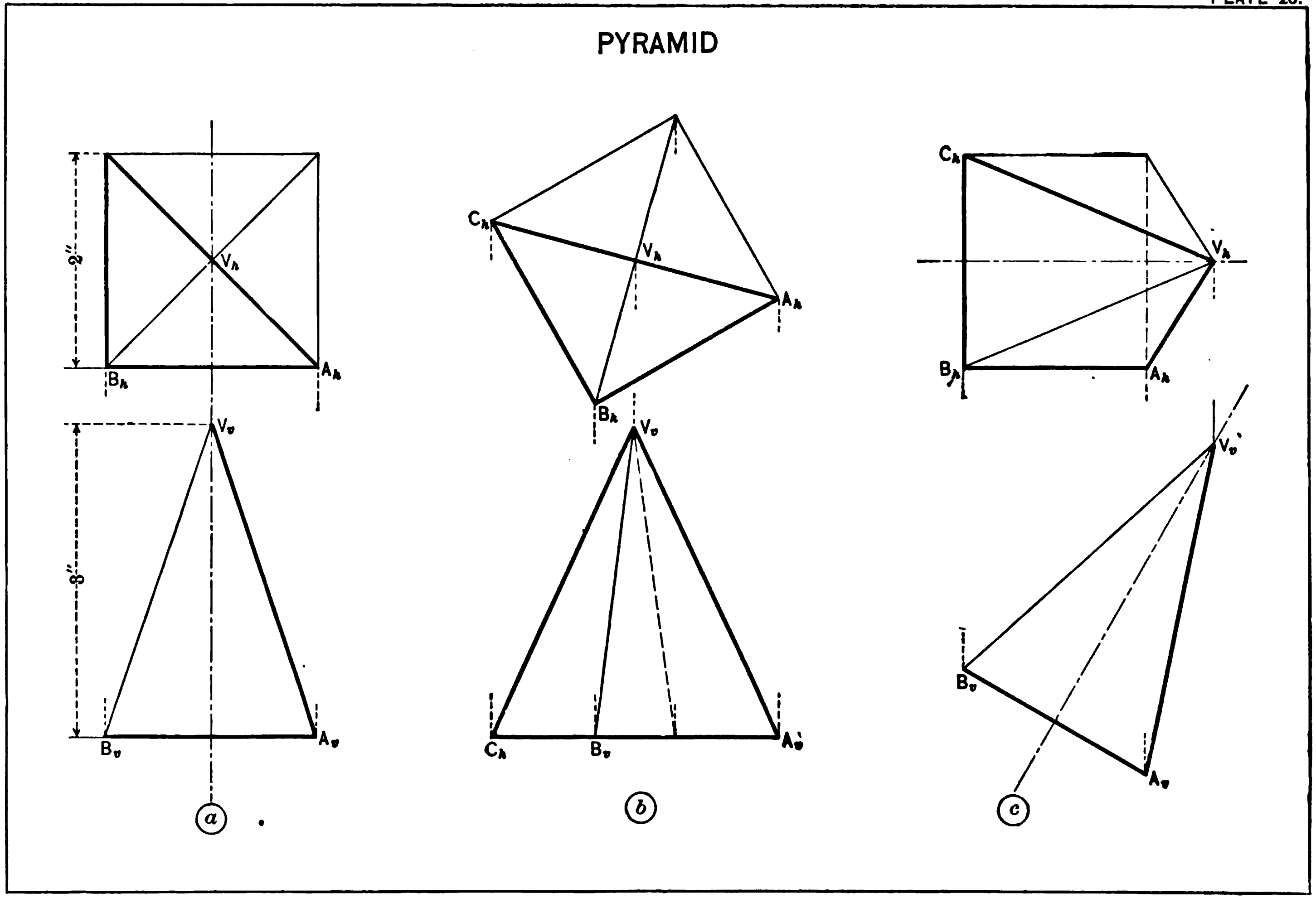

PYRAMID
2"
8"
V_h
B_h
A_h
V_v
B_v
A_v
(a)
C_h
V_h
A_h
B_h
V_v
C_h
B_v
A'_v
(b)
C_h
V_h
B_h
A_h
V'_v
B_v
A_v
(c)

PLATE 21.

44. **Section and Development of Pyramid.**—Draw the projections of a pyramid 3″ high, with a square base whose side is 2″; cut the pyramid by a section plane, show the section, make a projection on an S plane parallel to the section, develop the pyramid, and trace on the pattern the lines of intersection of the pyramid and section plane.

(*a*) The base of the pyramid is parallel to H, and V is inclined to a side of the base at an angle of 30 degrees.

In the projection on H, the center of the base is at (6.75″, 6″).

In the projection on V, the middle of the base is at (6.75″, 1.25″).

The section plane is perpendicular to H, and makes an angle of 45 degrees with V, $C_h V_h = \frac{7}{8}$″.

To find C_v and D_v, the projections on V of the points where the edges are cut by the section plane, C_h and D_h may be projected down to C_v and D_v respectively. Because the angle formed by the projecting line with the projection of the edge is so small as to make the intersection not sharp or definite, another and better method is to represent an auxiliary or supplementary projection on a plane parallel to both the axis and a diagonal of the base. One-half of such a projection is shown at (*d*). It is a right triangle one side of which, $f\,m$, is one-half the diagonal of the base, $V_h\ G_h$, and the other side the altitude of the pyramid. $f\,v$ is at $x = 10$″, and $f\,m$ at $y = 4.5$″. The distance $f\,l = V_h C_h$, then $l\,C_s$ = the perpendicular distance from C_v to the projection of the base $A_v E_v$; $f\,k = V_h D_h$, and $k\,D_s$ = the perpendicular distance from D_v to the projection of the base.

(*b*) Draw the projection on an S plane that is parallel to the section plane to show the section in its true size and form.

In the projection on S, V_s is at (8.5″, 4.25″).

$A_s E_s$ is parallel to $A_h E_h$. The perpendicular distance from C_s to the base $A_s E_s$ is equal to $l\,C_s$ in (*d*), and the perpendicular from D_s to $A_s E_s = k\,D_s$ in (*d*).

(*c*) In the development, V is at (4.25″, 4.25″).

With V as a center and the true length of an edge of the pyramid as a radius an arc is described. On this arc, each way from *m*, the sides of the base are marked off as chords of the arc. $A\,N = A_h N_h$; $E\,G = E_h G_h$; $V\,C = v\,C_s$ in (*d*), which is the true length of the edge from the apex to the point where the edge is cut off by the section plane at C, and $V\,D = v\,D_s$.

SECTION AND DEVELOPMENT OF PYRAMID

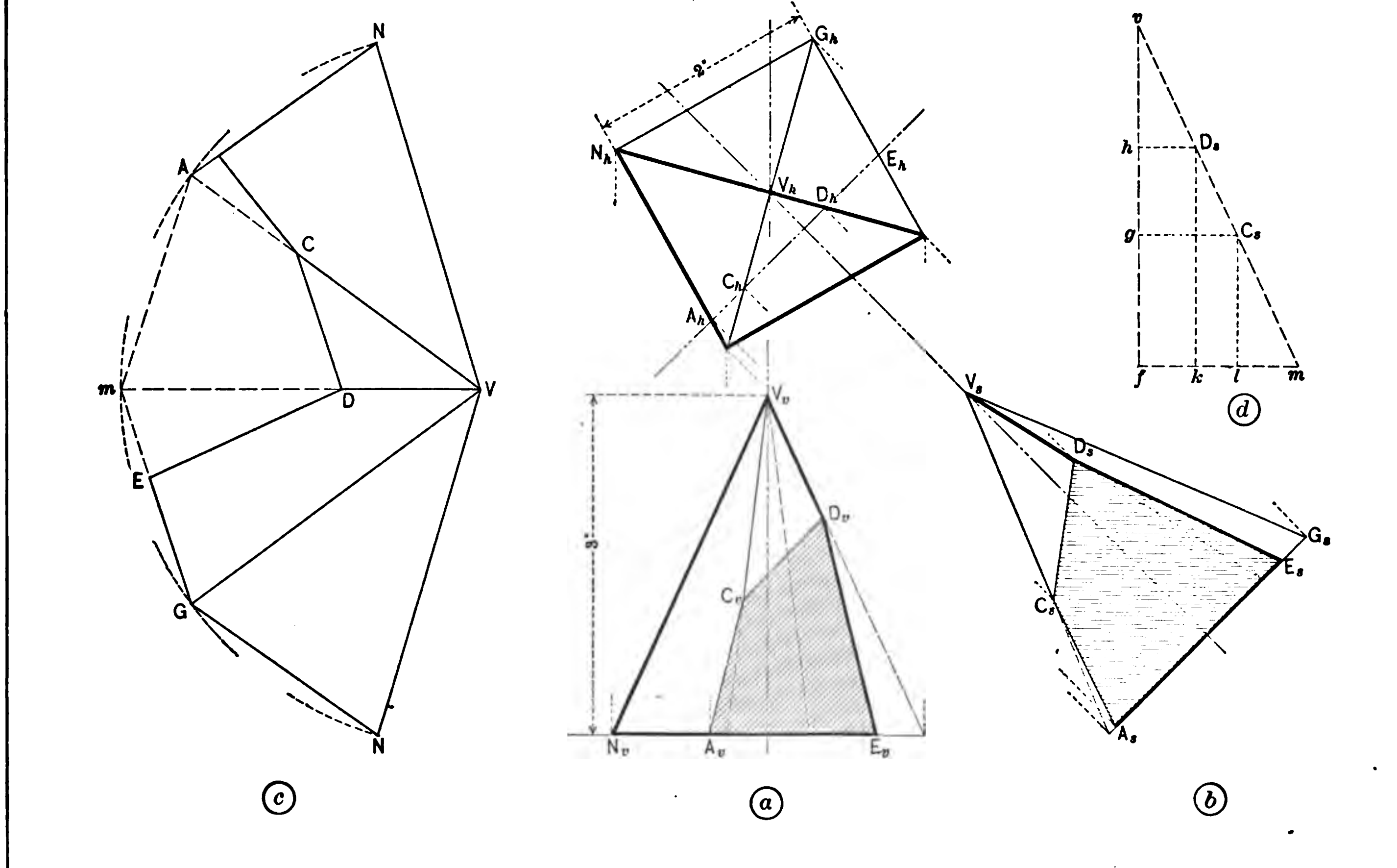


PLATE 22.

45. **Hexagonal Pyramid.**—Draw the projections and development of a hexagonal pyramid with dimensions as shown.

(*a*) The base is parallel to H, and V is parallel to a side of the base.

In the projection on H, the center of the base is at (2.75″, 6″).

In the projection on V, the center of the base is at (2.75″, 1.25″).

A section plane, which intersects the axis in the projection on V at (2.75″, 3.25″), is passed perpendicular to V and makes an angle of 45 degrees with H. Show the section in the projection on H.

(*b*) Draw a projection on P to the right.

In the projection on P, the center of the base is at (6.25″, 1.25″). Show the section on this projection.

(*c*) Develop the lower part of the pyramid below the section.

The true length of the radius of the arc is equal to $V_v E_v$, since this edge of the pyramid is projected on V in its true length, being parallel to V.

The distances V D, V C, etc., are equal to the true length of the edges from the apex to the points where they are cut by the section plane, that is, $V D = V_v D_v$, $V C = V_v f$, etc.

HEXAGONAL PYRAMID

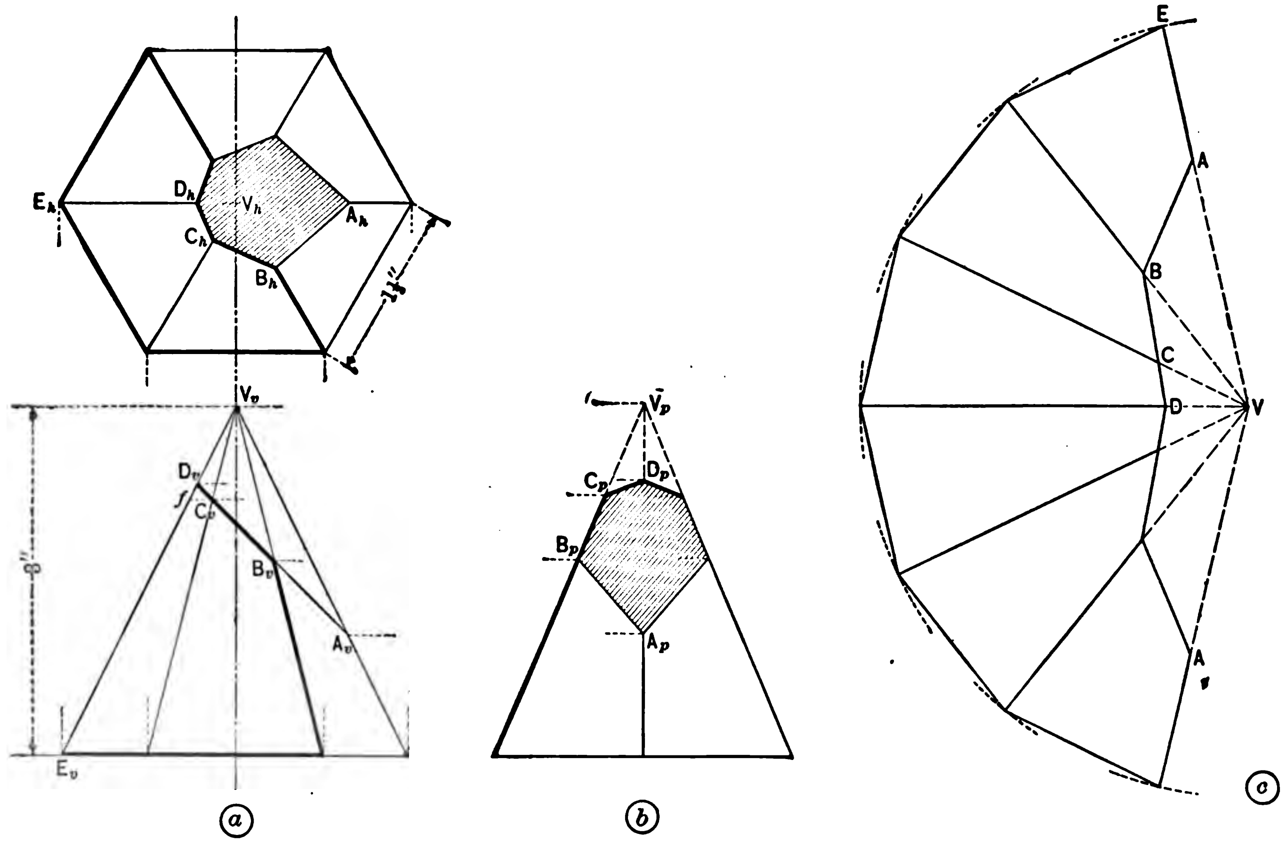


PLATE 23.

46. Intersection of Pyramid and Prism.—With dimensions as shown, draw the projections of a hexagonal pyramid and a square prism intersecting; develop the pyramid, and trace on the pattern the lines of intersection of the two surfaces.

(*a*) The pyramid has the same position as in the previous plate. The prism has a diagonal of its base parallel to H, and the diagonal is equal to the edge of the base of the pyramid. V is parallel to the plane of its base.

In the projection on H, the center of the base is at (2.75″, 4.5″).

In the projection on V, the center of the base is at (2.75″, 2.5″).

Draw the projection of the prism first on V, then on H.

B_h and D_h are found by projecting up from B_v and D_v respectively, or the distance from B_h to the axial line = the distance from B_v to the axial line, etc. C_h is found as follows: Conceive a plane to be passed parallel to the base of the pyramid, cutting from its surface a hexagon similar to the base. In the projection on V, F_v is the point where this plane cuts one edge of the pyramid, F_h is its projection on H. A line drawn through F_h parallel to the side of the base is the projection on H of the side of the hexagon cut from that face by the auxiliary plane. The point where the edge of the prism meets the projection of that line on H is C_h. Similarly A_h and E_h may be found.

(*b*) Draw the projection of these two surfaces on P to the right.

In the projection on P, the center of the base of the pyramid is at (6.25″, 1.25″).

$C_p\,G_p = C_h\,G_h$. In the projection on H, A_h and E_h may be found more conveniently and accurately from this projection than by the method given above, thus: $A_h\,K_h = A_p K_p$ and $E_h\,K_h = E_p\,K_p'$.

(*c*) In the development of the pyramid, V is at (11.5″, 4.25″). The front face is placed symmetrically with the horizontal line in the middle of the plate.

$V\,F = V_v\,F_v$, $C\,F$ is parallel to the line of the base of that face and equals $C_h\,F_h$; $A\,V = A_p\,V_p$, $E\,V = E_p\,V_p$.

$B\,V = b\,V_v$ and $D\,V = d\,V_v$ the true lengths, respectively, of the edges of the pyramid from the apex to the points B and D.

INTERSECTION OF PYRAMID AND PRISM

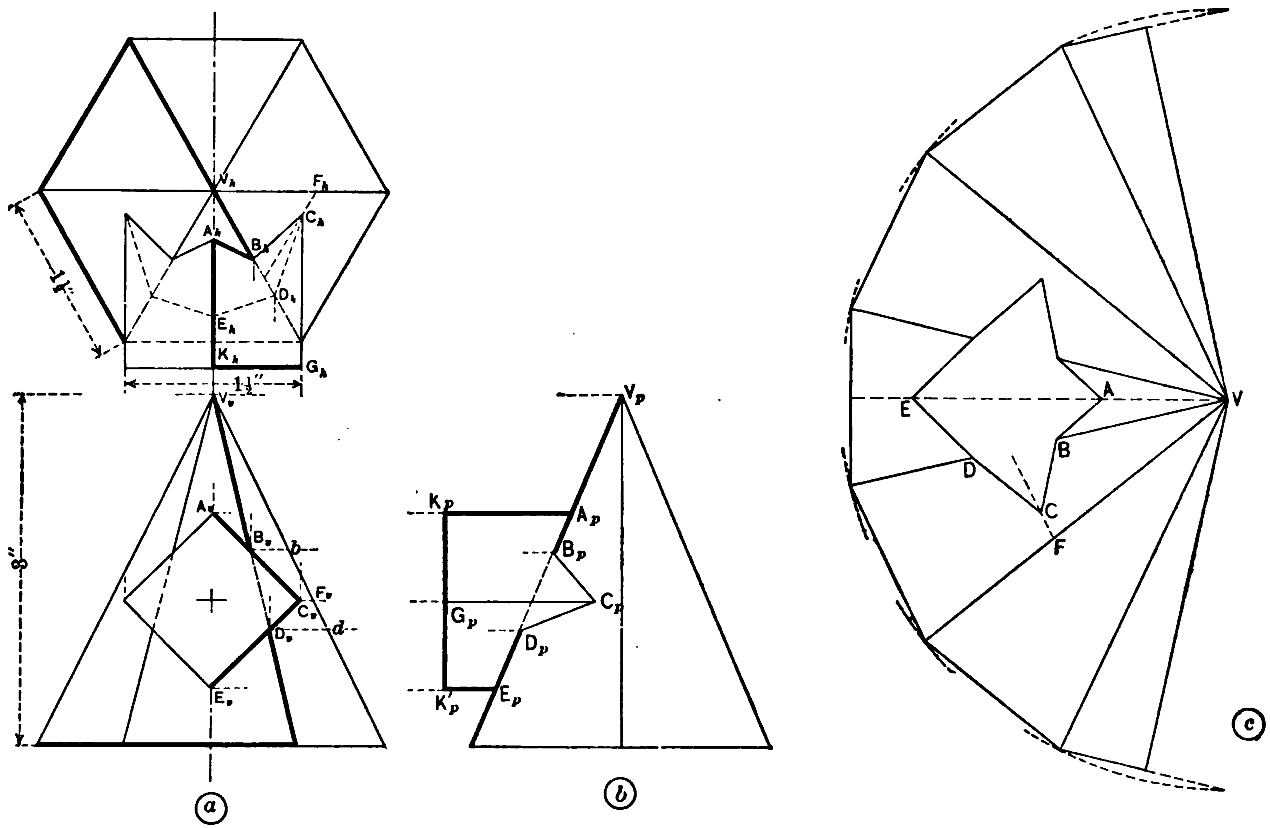


PLATE 24.

47. **The Cylinder.**—Draw the projections of a right cylinder. The radius of the base is 1″ and the height of the cylinder is 3″.

(*a*) When the axis is perpendicular to H.

In the projection on H, the center of the base is at (2″, 6″).

In the projection on V, the center of the lower base is at (2″, 1″).

(*b*) When the axis makes an angle of 30 degrees with H, and V is parallel to the axis.

In the projection on H, the center of the lower base is at (4.5″, 6″).

In the projection on V, the center of the lower base is at (4.5″, 1.75″).

Draw first the projection on V, then a projection on a supplementary plane which is parallel to the base and one-quarter of an inch from it. Show one-half of the projection of the base.

To obtain the projections on H of the bases, points on the line $E_v B_v$ are projected up to the line $D_h C_h$, then $C_h B_h = C_s B_s$, $D_h E_h = D_s E_s$, etc.; or the two axes of the ellipse of the base being known, the ellipse may be constructed on the axes. The bases are equal in their projections.

(*c*) Where the axis is perpendicular to H.

In the projection on H, the center of the base is at (9.75″, 6″).

In the projection on V, the center of the base is at (9.75″, 1″).

A section plane is passed cutting the axis in the projection on V at A_v (9.75″, 2″), making an angle of 30 degrees with H, and is perpendicular to V. Show the section on the projection on H.

Draw on a supplemental plane a projection of the ellipse cut from the surface by the section plane, so as to show the section in its true size and form. The supplementary plane is parallel to the section plane and 1.25″ from it.

$A_s C_s = A_h C_h$, $B_s D_s = B_h D_h$, etc., or the ellipse may be constructed on its major and minor axes.

THE CYLINDER

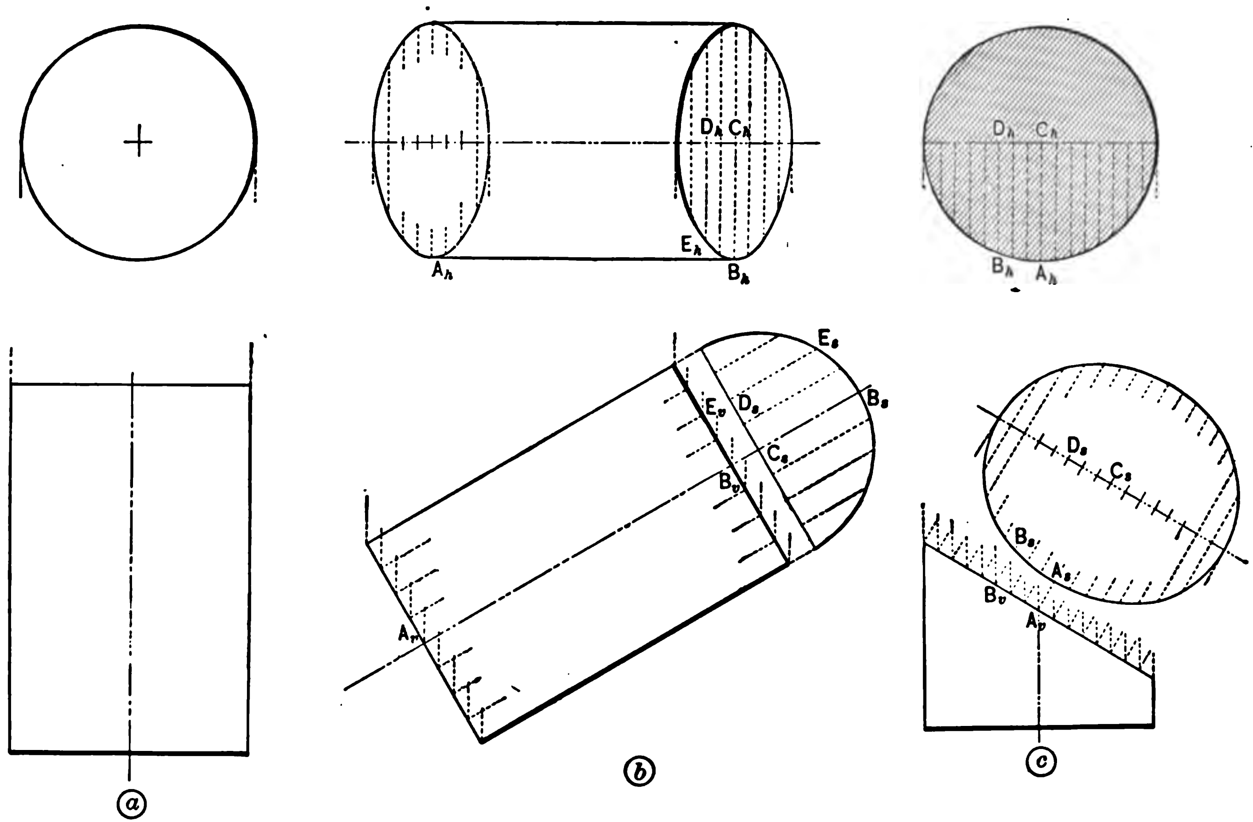


PLATE 25.

48. Section and Development of Cylinder (A).—Draw the section of a right cylinder and develop it, tracing on the pattern the outline of the section. The diameter of the base is 2″ and the height of the cylinder 3″.

(*a*) The axis of the cylinder is parallel to both H and V. In the projection on H, the lower end of the vertical diameter of the left base, F_h, is at (1.5″, 5.5″).

In the projection on V, F_v is at (1.5″, 1.5″).

Project one-half of the right base on an S plan, which is parallel to this base. C_s is at (4.75″, 2.5″).

Pass a section plan cutting the axis in the projection on V at a point 3.8″ from the vertical axis of the plate, that is, at C_v. The section plane is perpendicular to V and makes an angle of 45 degrees with H. Show the section in the projection on H. $A_h C_h = A_s C_s$, $B_h D_h = B_s D_s$, etc.

Such a section will always cut the whole or part of the curve of an ellipse (in this instance an arc of a circle, the circle being one limit of the ellipse) from the surface of the cylinder. $A_h C_h = A_s C_s$ = one-half the minor axis of the ellipse, and $H_h G_h$ = the projection of $H_v G_v$ on a horizontal line. Instead of determining the points on the curve of the ellipse by the method indicated, the ellipse may be constructed on its axes.

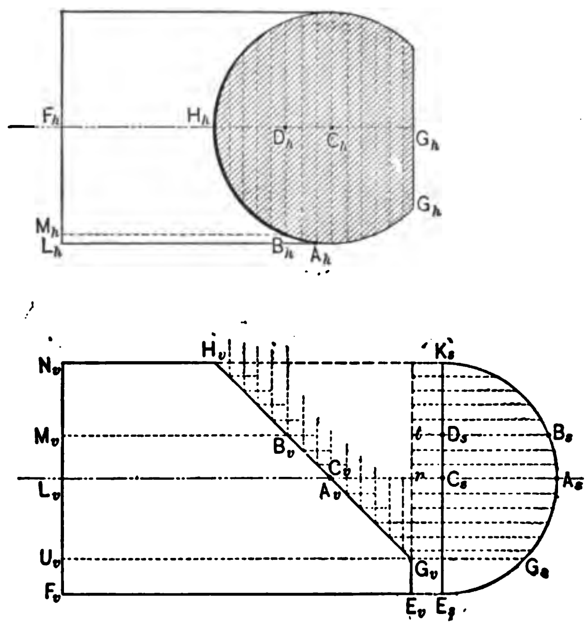
(*b*) The cylinder when developed will be a rectangle, the longer side equal to the rectified * base, and the shorter side equal to the height of the cylinder. The point E of the pattern is at (11″, 4.25″).

To trace the outline of the section on the pattern, the distance EG = the arc $E_s G_s$ rectified. The divisions of the line EG, produced each way from E, are equal to the corresponding divisions on the semi-circumference from G_s to K_s rectified. $AL = A_v L_v$, or $AR = A_v r$, $BM = B_v M_v$, or $BT = B_v t$, etc.

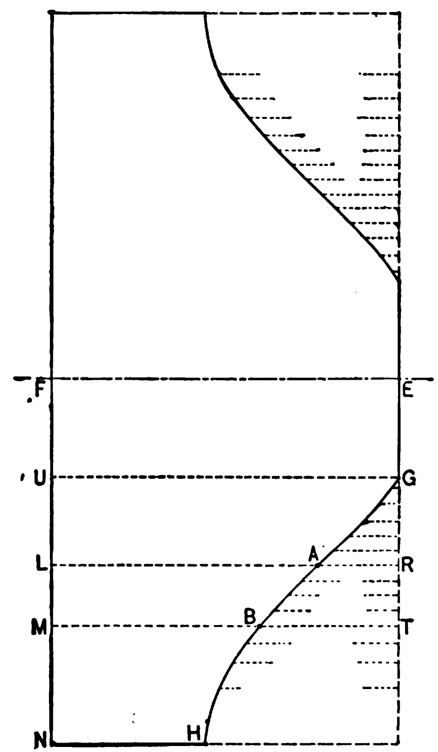
* To rectify an arc or curve is to lay out on a straight line a distance equal to the length of the arc or curve. Divide the curve into spaces so small that the length of the curve and its chord for each space shall not differ materially in length. Lay off on the straight line in succession spaces of the same length as those on the curve.

SECTION AND DEVELOPMENT OF CYLINDER

(A)



(a)



(b)

PLATE 26.

49. Section and Development of Cylinder (B).—Draw the projections of a right cylinder whose height is 3″, and the diameter of whose base is 2″, cut it by a section plane, develop the cylinder, and trace on the pattern the outline of the section.

(*a*) The axis of the cylinder makes an angle of 30 degrees with H, and V is parallel to the axis.

In the projection on H, the center of the base, G_h, is at (2″, 6″).

In the projection on V, the center of the base, G_v, is at (2″, 1.75″).

Draw the projection on V first, then find the projections of the bases on H as in (*b*) of Plate 24.

Pass a section plane parallel to H, the edge of which in the projection on V is 2.75″ above the horizontal axis of the plate. Show the section in the projection on H. To do this, draw one-half of the projection of the cylinder on an S plane parallel to the right base and one-fourth of an inch from it. $A_h L_h =$ twice $a A_s$, $E_h H_h = C_s E_s$, $F_h M_h = f F_s$, etc.

(*b*) In the development of the cylinder, let the line D B, which is equal in length to $D_v B_v$, the height of the cylinder, be 4.25″ above, and parallel to, the horizontal axis of the plate. Let B be at $x = 11''$. As in the previous plate, the pattern is a rectangle, one side of which is equal to the height of the cylinder and the other side equal to its base rectified. A B = arc $A_s B_s$ rectified, and the space on the line A *f*, produced, are equal respectively to the corresponding spaces on the arc $A_s E_s F_s$ rectified. G E = $G_v E_v$, or E *e* = $E_v K_v$, F N = $F_v N_v$, or F *f* = $F_v Q_v$, U R = $U_v R_v$. The two parts of the pattern on either side of the line B D are equal.

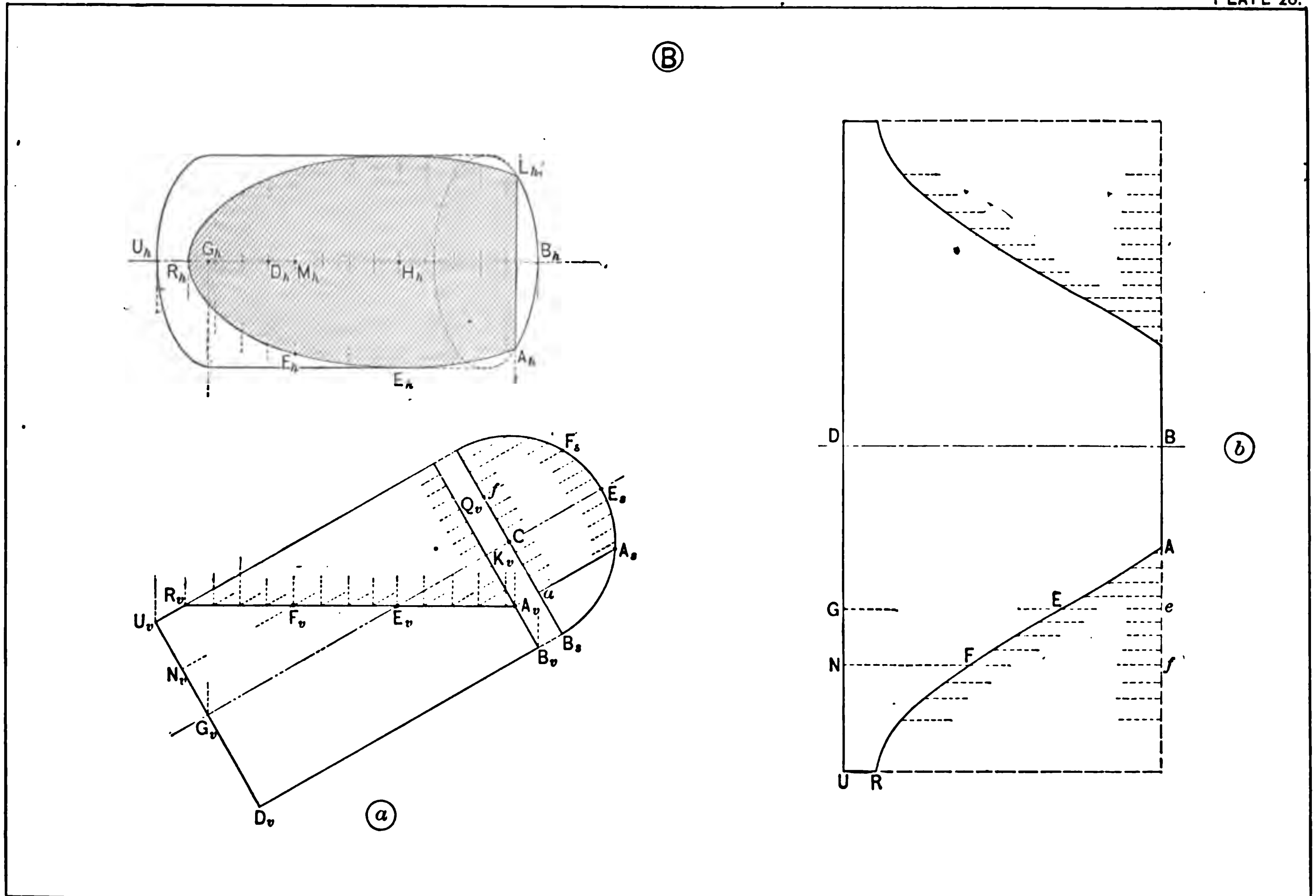

B
U_h
G_h
R_h
D_h
M_h
H_h
B_h
L_h
A_h
F_h
E_h
F_s
E_s
A_s
B_s
f
Q_v
C
K_v
a
R_v
U_v
F_v
E_v
A_v
B_v
N_v
G_v
D_v
a
D
B
b
A
E
G
e
F
N
f
U
R

PLATE 27.

50. Intersection of Prism and Cylinder.—Draw the projections of a square prism and a right cylinder which intersect and develop each, tracing on the patterns the outline of the intersection.

The cylinder is 3″ in height, and the diameter of its base is 2″. The diagonal of the base of the prism is 1.5″.

(*a*) The axis of the cylinder is perpendicular to H, and the axis of the prism is perpendicular to V. The axes intersect.

In the projection on H, the center of the upper base of the cylinder is at (2.5″, 6″).

In the projection on V, the center of the upper base of the cylinder is at (2.5″, 4″).

In the projection on H, the center of the base of the prism is at (2.5″, 4.5″).

In the projection on V, the center of the base of the prism is at (2.5″, 3″).

(*b*) Draw the projections of the two surfaces on the P plane to the right.

In the projection on P, the center of the upper base of the cylinder is at (6″, 4.5″).

In the projection on P, the center of the base of the prism is at (4.5″, 3″).

$D_p\,G_p = G_h\,D_h$, or $L_p\,D_p = L_h\,D_h$, $C_p\,E_p = C_h\,E_h$, or $M_p\,C_p = M_h\,C_h$, $N_p\,F_p = N_h\,F_h$. or $P_p\,N_p = P_h\,N_h$, etc.

(*c*) Develop the front half of the cylinder, tracing on the pattern the intersection of the prism with it.

The middle line, C U, is parallel to, and 2.25″ from, the horizontal axis of the plate. The left side is parallel to, and 8″ from, the vertical axis of the plate.

A B = one-half the circumference of the base of the cylinder rectified; the other side of the pattern equals the height of the cylinder.

$B\,R = B_h\,D_h$ rectified; the spaces on B R produced are equal respectively to the corresponding, rectified spaces on the arc $D_h\,N_h\,C_h$. $R\,D = R_v\,D_v$, $Q\,N = Q_v\,N_v$, etc. $N\,S = N_v\,S_v$, $C\,U = C_v\,U_v$, etc. The part below the middle line of the pattern is equal to the part above.

(*d*) Develop the prism and trace on the pattern its intersection with the cylinder. The lower line of the pattern is parallel to, and 5.5″ from, the horizontal axis of the plate, and the middle line, C M, is 8″ from the vertical axis of the plate. $M\,L = C_v\,D_v$, $M\,P = C_v\,N_v$, etc., $M\,C = M_h\,C_h = M_p\,C_p$, $D\,L = D_h\,L_h = D_p\,L_p$, $N\,P = N_h\,P_h = N_p\,P_p$, etc.

INTERSECTION OF PRISM AND CYLINDER

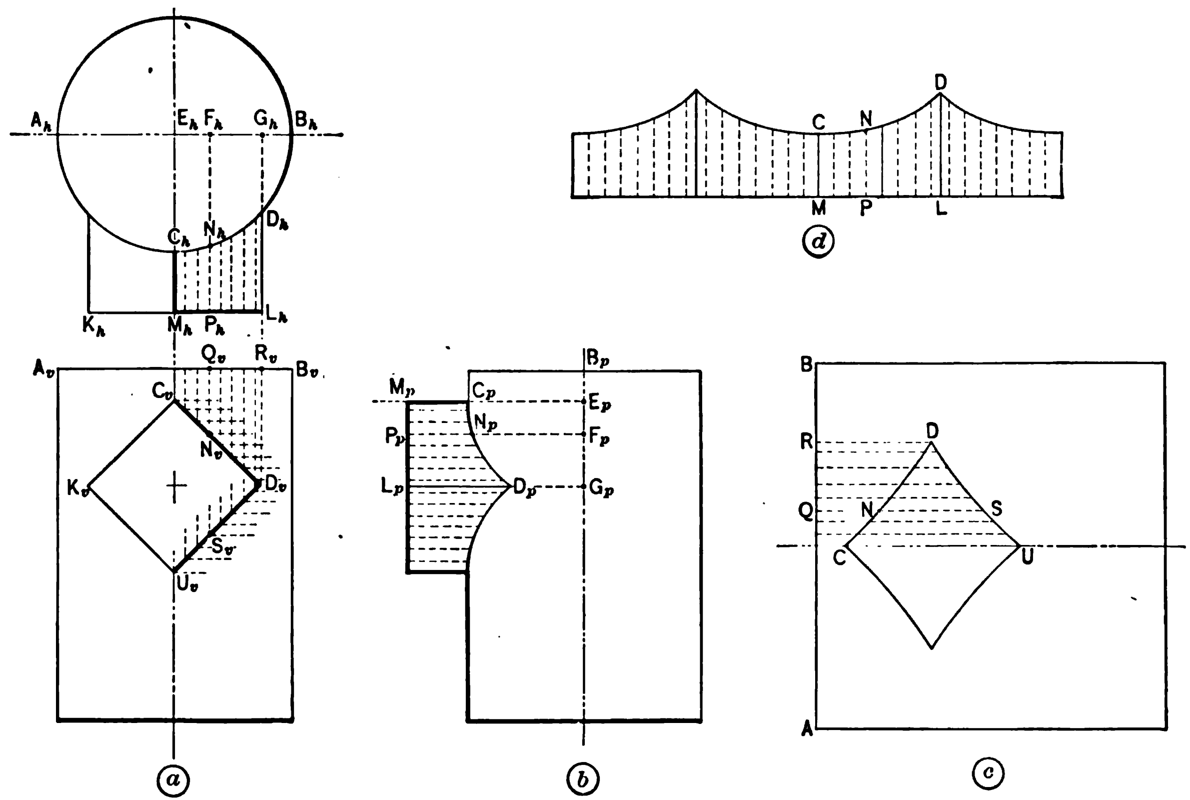


PLATE 28.

51. **Intersection of Two Cylinders.**—Draw the projections of two intersecting right cylinders, with their axes intersecting at right angles; develop each, and trace on the patterns of each its intersection with the other.

(*a*) The axis of one cylinder (for convenience called A) is perpendicular to H, it is 3″ long, and the base of the cylinder has its diameter 2″. The axis of the other cylinder (B) is perpendicular to V, and the diameter of its base is 1.5″.

In the projection on H, the center of the upper base of the A cylinder is at (2.5″, 6″).

In the projection on V, the center of the upper base is at (2.5″, 4″).

In the projection on H, the center of the base of the B cylinder is at (2.5″, 4.5″).

In the projection on V, the center of the base is at (2.5″, 3″).

(*b*) Draw the projection of these cylinders on a P plane to the right.

In the projection on P, the center of the upper base of the A cylinder is at (6″, 4″).

In the projection on P, the center of the base of the B cylinder is at (4.5″, 3″).

The arc of the base of the B cylinder which is projected on P in $M_p\,P_p$ is projected on V in $C_v\,N_v$ and on H in $M_h\,P_h$.

$C_p\,E_p = C_h\,E_h$, or $C_p\,M_p = C_h\,M_h$; $N_p\,F_p = N_h\,F_h$, or $N_p\,P_p = N_h\,P_h$; $D_p\,G_p = D_h\,G_h$, or $D_p\,L_p = D_h\,L_h$, etc.

The projection on P of the lower part of the curve of intersection is the same as the upper half inverted.

(*c*) Develop the front half of the A cylinder. The front half of the upper base rectified lies in the line A B which is parallel to the vertical axis of the plate and 8″ from it. The middle element of the cylinder between A and B is parallel to the horizontal axis of the plate and 2.5″ from it.

The divisions on the line A B are the corresponding rectified arcs between C_h and D_h, that is, Q R $= N_h\,D_h$ rectified. R D $= R_v\,D_v$, Q N $= Q_v\,N_v$, etc. C V $= C_v\,V_v$, N U $= N_v\,U_v$, etc. The half of the curve below the line C V is the same as that above the line inverted.

(*d*) Develop the B cylinder. The base is rectified on the line M L, produced each way, which is parallel to the horizontal axis of the plate and 5.5″ from it; the middle line M C is parallel to the vertical axis of the plate and 8″ from it.

The divisions of the base between M and E are equal to the corresponding divisions, rectified, on the circle between M_v and L_v in the projection on V. M C $= M_p\,C_p = M_h\,C_h$, N P $= N_p\,P_p = N_h\,P_h$, L D $= L_p\,D_p = L_h\,D_h$, etc. The curve to the right of D L is the same as that to the left reversed.

INTERSECTION OF TWO CYLINDERS

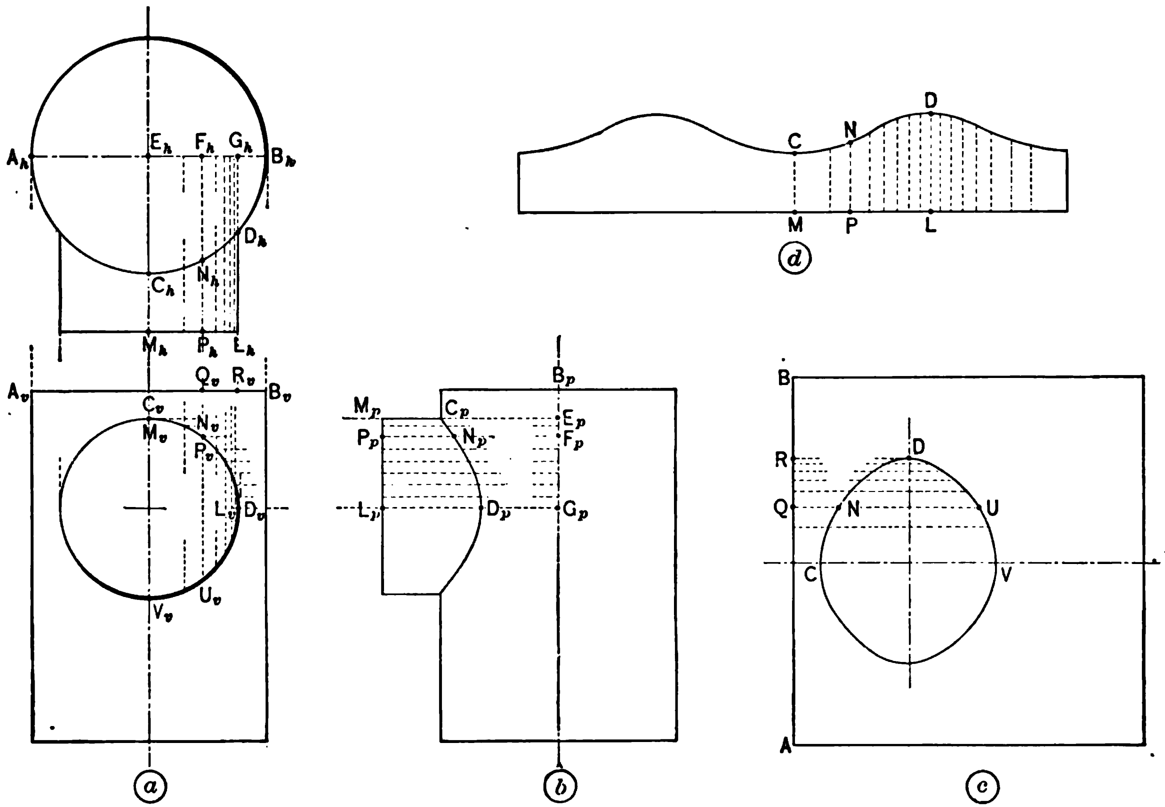


PLATE 29.

52. Intersection of Cone and Prism.—Draw the projections of a right cone, 3″ high, with the diameter of its base 2.5″, intersecting a hexagonal prism with the diagonal of its base 1.25″, their axes intersecting at right angles. Develop both surfaces and trace on the pattern of each its intersection with the other.

(*a*) The axis of the cone is perpendicular to H, and the axis of the prism is perpendicular to V.

In the projection on H, the apex of the cone, V_h, is at (2.5″, 6″).

In the projection on V, V_v is at (2.5″, 4″).

In the projection on H, the center of the base of the prism is at (2.5″, 4.5″).

In the projection on V, the center of the base of the prism is at (2.5″, 2.25″).

Draw the projection of the prism on V first.

Assume planes to be passed perpendicular to the axis of the cone. These planes will cut from the surface of the cone circles, and from the prism lines parallel to its edges. For convenience of construction pass these planes at regular intervals, dividing the line $D_v\ E_v$, in the projection on V, into 12 equal spaces. With V_h as a center and $V_h\ D_h = D_v\ a$ as a radius describe the arc $K_h\ D_h$ produced. This arc is the projection on H of the intersection of the cone with the upper face of the prism; and K_h, the point where this arc meets the edge of the prism, $Q_h\ K_h$, in the projection on H, is the intersection of that edge with the surface of the cone. Similarly the arc $F_h\ E_h$ produced, and the point F_h, are found, $E_v\ b = V_h\ E_h$. Divide $Q_h\ M_h$, in the projection on H, into six equal parts (or project the points K_v, G_v, etc., up to Q_h, M_h, etc.); also divide the distance $D_h\ E_h$ into 12 equal spaces, and through the points of division draw arcs with V_h as a center (or obtain the lengths of the several radii from the projection on V, as $d\ e$, $c\ f$, etc.). These arcs are the projections on H of the arcs of circles cut from the cone by the 12 planes passed perpendicular to the axis. The projections on V of these arcs lie in the straight lines which would, if drawn, be perpendicular to the line $D_v\ E_v$ and divide it into 12 equal spaces.

In the projection on H where these arcs meet, the lines cut from the prism by each plane respectively will give points on the curved lines of intersection $K_h\ G_h$ and $G_h\ F_h$.

(*b*) Draw the projection of these surfaces on a P plane to the right.

In the projection on P, the apex of the cone, V_p, is at (6″, 4″).

$Q_p\ D_p = L_h\ D_h$, or $D_p\ g' = V_h\ D_h$; $Q_p\ K_p = Q_h\ K_h$, or $K_p\ g' = K_h\ g$; $S_p\ R_p = S_h\ R_h$, or $R_p\ n' = R_h\ n$, etc.

(*c*) In developing the cone the apex, V, is at (8″, 4.25″), VB is horizontal, the arc A B, produced, is drawn with a radius equal to $V_v\ A_v$, with the center at V. Its length equals the rectified circumference $A_h\ B_h\ A_h$. With radii equal to $V_v\ a$, $V_v\ c$, $V_v\ f$, etc., respectively, draw 13 concentric arcs in the pattern. These arcs are the developed arcs cut from the surface of the cone by the planes passed perpendicular to its axis, as they are measured off in their true lengths.

Lay out on the arc D K the length of the arc $D_h\ K_h$ on either side of the center line D E, and on E F the length of $E_h\ F_h$. Similarly lay off on either side of the center line D E, on the several arcs, the lengths of the corresponding arcs in the projection on H respectively.

(*d*) In developing one-half of the prism, lay out the sides comprising half the base on a line parallel to the horizontal axis of the plate and 5.5″ from it. The middle line, D L, of the lower face is parallel to the vertical axis of the plate and 5″ from it.

$L\ D = Q_p\ D_p$, $g''\ D = g'\ D_p = V_h\ D_h$. With g'' as a center and $V_h\ D_h$ as a radius describe the arc D K.

$Q\ K = Q_p\ K_p$, $S\ R = S_p\ R_p$, etc.

$E\ h'' = E_p\ h'$, or $V_h\ E_h$.

With h'' as a center and $V_h\ E_h$ as a radius draw the arc E F. $E\ L = E_p\ L_p = E_h\ L_h$, $F\ N = L_p\ F_p$, etc.

One-half the pattern of the prism is shown; the other half is the same figure reversed on the other side of either of the lines E L or D L.

INTERSECTION OF CONE AND PRISM

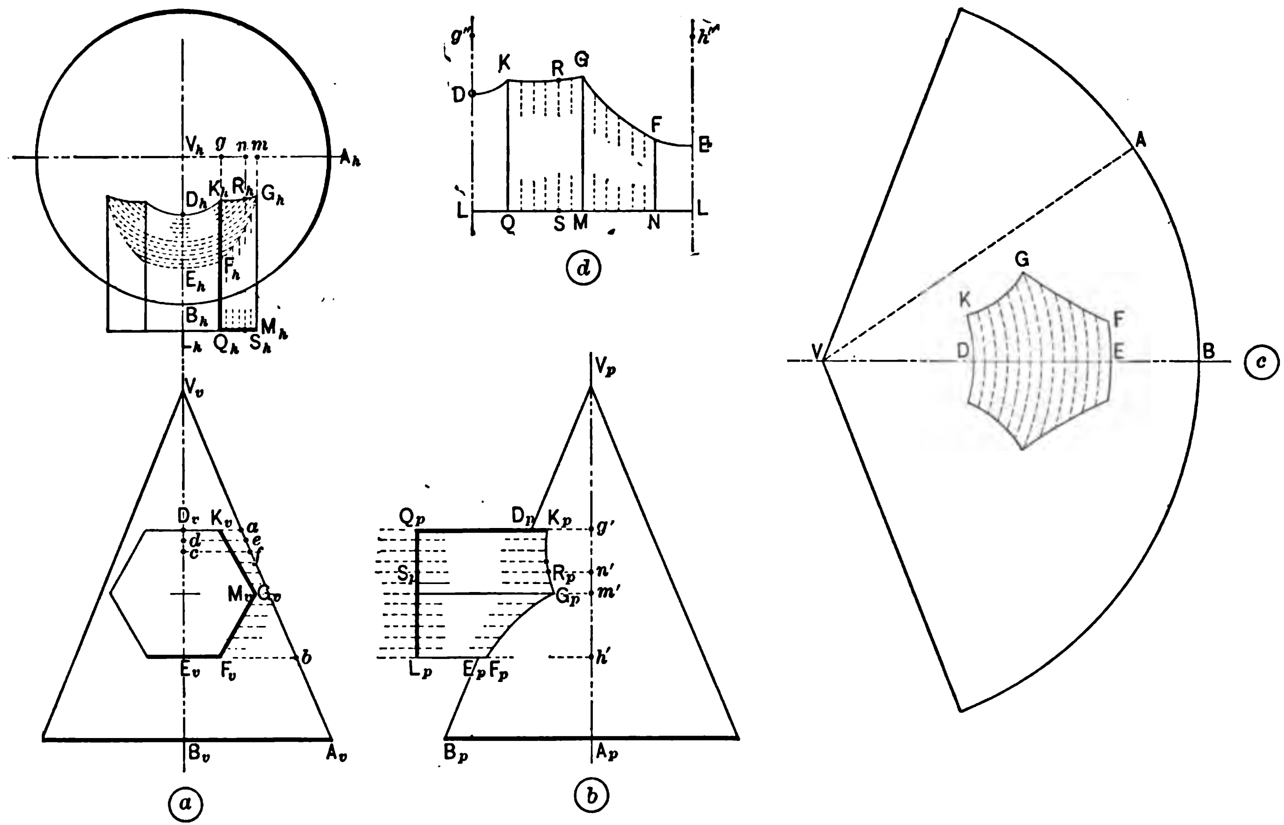


PLATE 30.

53. **Cone and Oblique Cylinder.**—Draw the projections of a right cone, with its axis vertical, intersected by a cylinder whose right section is a circle, and whose axis is parallel to P and makes an angle of 45 degrees with both H and V. The height of the cone is 3.25″, and the diameter of its base 2.5″. The diameter of the base of the cylinder is 1.5″.

(*a*) In the projection on H, the apex of the cone V_h is at (6.25″, 6.25″).

In the projection on V, V_v is at (6.25″, 4.5″).

After drawing the projections of the cone on H and V, next draw its projections on P, and also draw the base and the axial line of the cylinder on P.

(*b*) In the projection on P, V_p is at (10″, 4.5″).

In the projection on P, the center of the base of the cylinder is at (9″, 3″), and the axial line makes an angle of 45 degrees with a horizontal line.

Project the base of the cylinder on a supplemental plane parallel to it and 0.25″ from it, and draw the outside elements of the cylinder on P indefinitely prolonged to the right.

Now if horizontal planes are assumed to be passed through the cone and cylinder, they will cut unequal circles from the cone and equal ellipses (in whole or in part) from the cylinder. Pass 12 or more of these planes equally distant apart. If the cylinder is supposed to be extended indefinitely to the right of the base, the cutting plane whose edge is E_p K_p will cut the ellipse from the cylinder shown at (*d*), the longer axis of which = E_p K_p, and the shorter axis equals the diameter of the base of the cylinder. The center of this ellipse is at (9.5″, 6″).

The plane whose edge is C_p *s* cuts from the cone a circle whose projection on H is an equal circle whose radius is V_h $s' = u$ s in the projection on P. This same plane cuts from the extended cylinder a part of an ellipse whose projection on H is equal to itself, one side of which is h' C_h. Where the circle and ellipse meet, at C_h, is a point lying on the surface of both the cone and cylinder and is therefore a point in their intersection. Similarly the other points in the projection on H are found, as L_h, F_h, M_h, etc.

If the ellipse E_s K_s be traced on vellum cloth and accurately cut out, the points where the ellipse meets the respective circles in L_h, C_h, F_h, etc., may be found in the following manner: For C_h lay off on the vertical axial line drawn through V_h the distance V_h $h' = u$ h in the projection on P, place the pattern of the ellipse so that K_s is on h' and the long diameter, K_s E_s, coincides with the vertical axial line, then the edge of the pattern meets the corresponding circle at C_h. V_h $m' = w$ m; here K_s is placed on m' and the edge of the pattern meets the corresponding circle at M_h; the radius of the corresponding circle $= w$ r.

Similarly the other points may be found, thus determining the curve of intersection in the projection on H in the points B_h. L_h, C_h, F_h, M_h, etc.

In the projection on P, C_p u is equal in length to the distance from C_h to the horizontal axial line in H, and the distance of C_v from the axial line in V is equal to the distance from C_h to the vertical axial line in H. Similarly the other points of the curve of intersection on P and V may be found.

The projections on H and V of the outline of the base of the cylinder may be found as in previous problems.

(*c*) A projection of these bodies is made on an S plane which is perpendicular to H and makes an angle of 60 degrees with V.

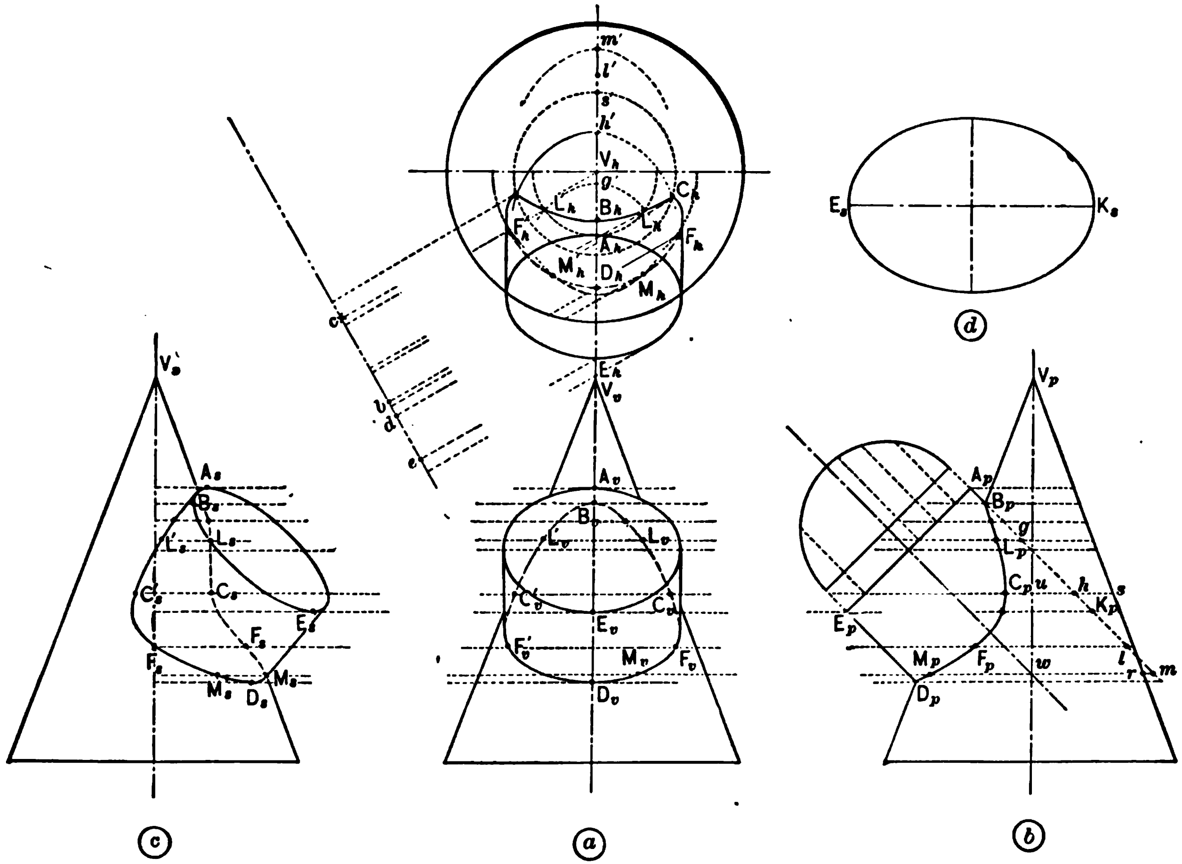
The line *e c*, produced, distant 2.5″ from V_h, shows the position of this plane. Instead of showing this projection on *e c* in this position, it is changed to another position in line with the bases of the projections of the cone in V and P, in order to utilize more conveniently the heights of the several points in those projections.

In the projection on S (in the second position), the apex of the cone, V_s, is at (2.5″, 4.5″).

To find M_s, first project V_h to *c* and M_h to *d*, then on the horizontal line through M_v lay off from the axial line through V_s the distance to $M_s = d$ c; on the horizontal line through F_v the distance from the axial line to $F_s = b$ c, *b* being the projection on *e c* of the point F_h in the projection on H. Similarly the other points of the line of intersection as projected on the S plane are found, as B_s, L_s, C_s, F_s, etc.

The points on the outline of the base of the cylinder are found in a similar way to that in which the points of the line of intersection are found.

CONE AND OBLIQUE CYLINDER



CHAPTER V.

ISOMETRIC AND OBLIQUE PROJECTIONS, SHADES, SHADOWS, AND PERSPECTIVE.

54. **Isometric Projection.**—A projection of an object may be made on one plane to show three dimensions (see Plate 11, (*c*)). In order that such a drawing may be useful as a working drawing, the lengths and true forms of the lines must be known. In an isometric drawing the principal lines, although not parallel to the plane of projection, are shown in their true lengths. If the plane of projection, or plane of the paper, is taken perpendicular to a body diagonal of a cube, the three adjacent edges of the corner to which the diagonal is drawn will be projected on this plane in lines which make equal angles with each other, viz., angles of 120 degrees. These lines are called the *isometric axes* of an *isometric projection*. One of the three axes is usually made vertical.

Since lines that are parallel to each other are projected in parallel lines, if the principal lines of an object are parallel to the three edges of a cube (which form a right trihedral angle), they may be projected parallel to the isometric axis.

The inclination to the plane of projection of the edges of the cube is 35 degrees 16 minutes nearly, then (by trigonometry) the projections of any definite portion of an edge, or any line parallel to it, will be equal to the length of that portion multiplied by the cosine of 35 degrees 16 minutes. An *isometric scale* may thus be made by taking any unit of measure on the object and making the unit of measure on the drawing equal to the cosine of 35 degrees 16 minutes times that unit. *But an isometric scale is quite unnecessary*, since all lines which are projected parallel to the isometric axes are equally foreshortened. *An ordinary scale may be used.*

Shade-lines.—In isometric projections the rays of light are assumed *parallel to the plane of projection and making with the horizontal an angle of 30 degrees;* the shade-lines are determined accordingly.

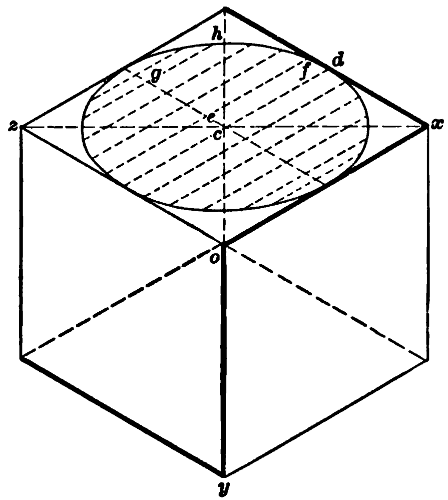
55. PLATE 31.

(*a*) Draw the isometric projection of a cube whose edge is 2″. *o x*, *o y*, and *o z* are taken as the isometric axes; these lines, each 2″ long, may be taken as the projections of the three adjacent edges of the cube. *o x* and *o y* each make an angle of 30 degrees with the horizontal. (Use the T square and 30-60 degree triangle.) In (*a*), *o* is at (3″, 4.25″).

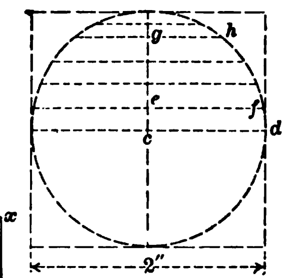
(*d*) Draw a circle in the upper base whose diameter is 2″. In (*a*), measure 1″ from *x* to *d*, draw *d c* parallel to *o x*. *c g* is parallel to *o z*. In (*d*), *c* is at (6″, 6″). Lay off *c g* in (*a*)=*c g* in (*d*). Draw *g h* in (*a*) parallel to *o x* and equal to *g h* in (*d*). Proceed in a similar manner to find other points.

(*b*) Draw the isometric projection of a vertical, hexagonal prism, 3½″ long, a side of whose base is 0.75″, intersected by a rectangular prism 1¾″ long, the edges of whose base are 1.25″ horizontal and 1″ vertical, and which is parallel to one face of the hexagonal prism. The upper face of the intersecting prism is 1″ below the upper base of the hexagonal prism. *o x*, *o y*, and *o z* may be taken as the axes or parallel to them. In (*b*), *c* is at (9″, 6″). *a b* in (*b*) is parallel to *o x*, and *h k* parallel to *o z*. *c k*=*c h* in (*b*) equals *c k*=*c h* in (*c*). *e z* in (*b*)=1″. *e g* in (*b*)=*e g* in (*c*), etc. In (*c*), *c* is at (6″, 2.25″).

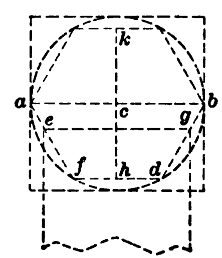
ISOMETRIC PROJECTION



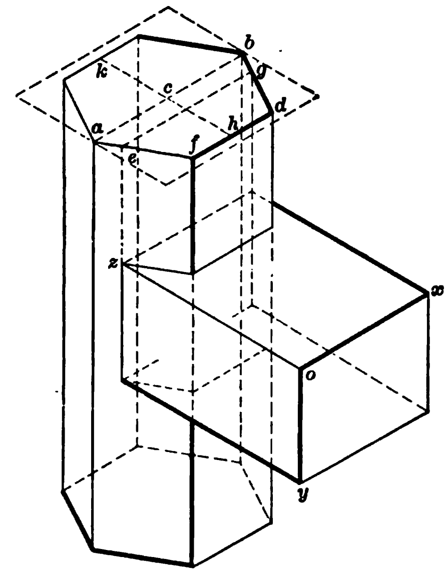
(a)



(d)



(c)



(b)

56. Oblique Projections. — All the preceding projections have been *orthographic*, that is, the projecting lines of the different points have been perpendicular to the plane upon which the drawing is made. *If the projecting lines are not perpendicular to this plane*, though parallel to each other, *the projections are called oblique.*

57. Cavalier Projections. — *When the projecting lines make an angle of 45 degrees with the plane of projection,* in any direction, the projection is oblique and is called a *cavalier projection.*

Cavalier projections are applicable, in general, to simple objects or bodies whose faces are at right angles to each other. One face is usually taken parallel to the plane of projection and so is shown in its true size; also the edges that are perpendicular to the plane of projection are shown in their true lengths, although the faces they bound are not shown in their true shape or size.

The advantage of cavalier projections for a working drawing is that the three dimensions are shown in one projection and the principal lines (those parallel and perpendicular to the plane of projection) are shown in their true lengths.

58. Pseudo-perspective. — *When the projecting lines make any angle less than 45 degrees with the plane of projection,* in any direction, the projection is oblique and is called *pseudo-perspective.* It resembles somewhat a true perspective which will be explained later. As in isometric and cavalier projections its application is limited to the projections of bodies made up principally of plane faces which are perpendicular to each other. One face is usually taken parallel to the plane of projection and (together with all other planes parallel to it) is shown in its true size, while the projections of the edges which are perpendicular to the plane of projection are foreshortened to any proportion of their true length, as one-half, two-thirds, etc., by taking the projecting lines at a corresponding angle to the plane of projection.

Shade-lines.—In oblique projections the rays of light are assumed to have the same direction as in orthographic projections, viz., the direction of the body diagonal of a cube (see Art. 34).

PLATE 32.

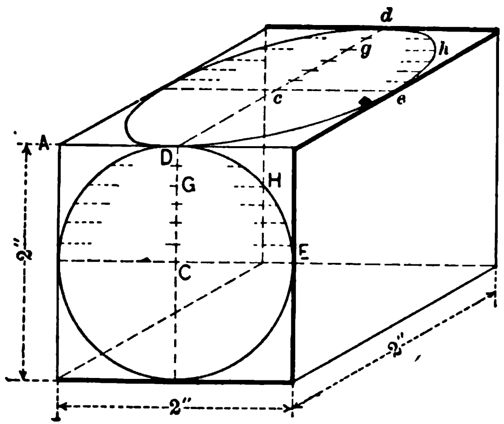
59. Draw the cavalier projections of a cube.

(*a*) The corner A is at (1.75″, 4.25″). The line D *d* makes an angle of 30 degrees with the horizontal (it may make any desired angle). The projection of the circle on the front face is a circle equal to the circle it represents. On the top face *c d* is divided into the same number of parts, as C D on the front face, then *g h*=G H, etc. *h* is a point in the circle, and other points are found in a similar way.

(*b*) Draw the pseudo-perspective of a cube. The corner A is at (8″, 4.25″). The line D *d* makes an angle of 30 degrees with a horizontal, and in this instance is made 1″ long or one-half the length of the line it represents. D *d* may have any desired direction and length, so long as the length is less than the line it represents. Points in the circumference of the circle on the top face may be found similarly to those in (*a*).

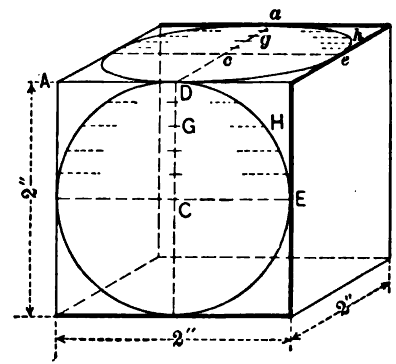
OBLIQUE PROJECTION

CAVALIER



(a)

PSEUDO-PERSPECTIVE



(b)

60. Shades and Shadows.—Regarding the sun as the source of light in determining shades and shadows, *the rays of light are practically parallel and may be so regarded.*

That part of a body which faces towards the source of light and upon which the rays of light fall is said to be *in the light*, and that part which faces away from it and upon which the rays do not fall is said to be *in the shade.*

The division-line between the light and shaded parts of a body is called the *line of shade.*

A *shadow* on any surface is that part of the surface from which the light is excluded by the intervention of some opaque body between it and the source of light. Parts of a body may cast shadows on other parts of the same body that are not in the shade, or upon other bodies or surfaces.

When one body casts a shadow on another, the *line of shade* casts the *outline of the shadow.*

The difference between a shade and a shadow is apparent. A shade is that part of any body or surface which is turned away from the light, and a shadow is that part of a body or surface from which the light is cut off by some opaque body.

To find the *projection of the line of shade* of a body which is made up of faces that are planes, the method of Art. 34 for determining which faces are in the light and which in the shade may be followed. If the body is made up of *curved surfaces*, it will be necessary to find the projection of rays of light which form an *enveloping*, or *tangent*, *surface;* the *line of tangency will be the line of shade.*

The *shadow of a point* on any surface is found by determining where a ray of light through the point pierces the surface; the projection of the shadow of the point is found by using the projections of the ray.

If a line is parallel to a plane, the shadow of the line on the plane will be parallel and equal to the line.

PLATE 33.

61. Find the projections of the shadow of a cube on a horizontal plane, also on a vertical plane which is parallel to V.

(*a*) Take the faces of the cube parallel to both H and V.

The horizontal plane (T) upon which the shadow is cast coincides with the bottom face of the cube, the trace of which on V is E_2 B_2. The vertical plane (R) is one-half an inch back of the cube, the trace of which on H is the horizontal line through B_1.

The edge of the cube is 2″. A_h is at (3″, 3.5″) and A_v is at (3″, 3″).

The line of shade of the cube, beginning at the bottom under A, follows the edge to A, thence along the top edge to C, thence to D, and thence to the bottom under D.

The projection on H of the ray of light through A is A_h A_1; its projection on V, A_v B_2. This ray pierces the T plane at a point whose V projection is B_2 and whose H projection is A_1. The V projection of the entire shadow on the T plane will fall in the line E_2 B_2; the projection on H of the outline of the shadow is A_h A_1 B_1 C_1 D_1 D_h. The line A_h A_1 is the projection of the shadow of the vertical edge of the cube through A, the line A_1C_1 that of the edge A C, C_1 D_1 that of the edge C D, and D_1 D_h that of the vertical edge through D.

That part only of the projection on V of the shadow of the cube on the R plane which is above the T plane is shown. The shadow of the vertical edge through A does not reach the R plane. That part of the edge A C from B to C casts the shadow which is projected on V in the line B_2 C_2, C_2 D_2 is the projection on V of the edge C D, and D_2 E_2 of a part of the vertical edge through D, both parallel and equal to edges casting the shadows.

The H projection of the entire shadow on the R plane falls in the horizontal line through B_1. The H projection of the shadow of the edge A C is A_1 C_1, equal and parallel to A C; also D_1 C_1 is equal and parallel to D C.

(*b*) A face of the cube is parallel to H, and another makes an angle of 30 degrees with V. The T plane is one-half an inch below, and parallel to, the bottom face of the cube. The trace of the R plane on H is the line through K_1 (11″, 7″) making an angle of 15 degrees with a horizontal.

E_h is at (7.5″, 4.25″), and E_v at (7.5″, 1.5″).

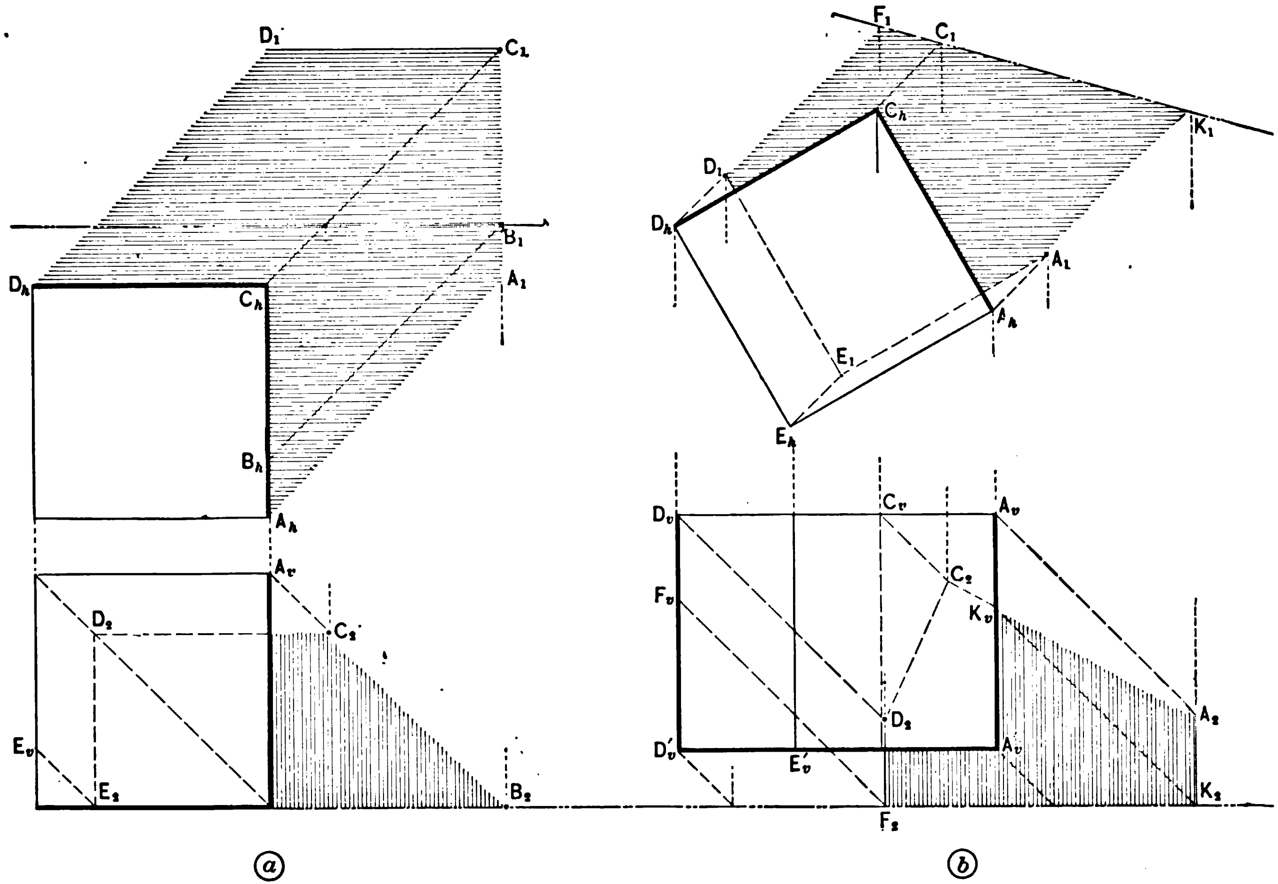
The line of shade follows in order the edges passing through the corners A′, A, C, D, D′, E′, A′.

The outline of the shadow on the T plane as projected on H is A_1 K_1 F_1 D_1 E_1 A_1, and the outline of the shadow on the R plane as projected on V, F_2 K_2 A_2 C_2D_2 F_2.

No part of the shadow of the cube on the T plane is shown beyond the R plane, and no part of the shadow on the R plane is shown below the T plane.

The section lines showing the projection of shadows should be drawn horizontal when the shadow is on a horizontal plane, and vertical when the shadow is on a vertical plane. The section-lines should be omitted when the shadow is hidden by the body in the projections.

SHADOWS



D_1
C_1
B_1
A_1
D_h
C_h
B_h
A_h
A_v
D_2
C_2
E_v
E_2
B_2
(a)
F_1
C_1
C_h
K_1
D_1
D_h
A_1
A_h
E_1
E_h
D_v
C_v
A_v
C_2
F_v
K_v
D_2
A_2
D_v'
E_v'
A_v
K_2
F_2
(b)

62. PLATE 34.

(*a*) Find the projections of the shaded parts of a cube and of its shadow on a horizontal plane (T) and on a vertical plane (R) parallel to V. Show only that part of the shadow on the T plane which is in front of the R plane, and that part of the shadow on the R plane which is above the T plane.

One edge of the cube is parallel to H, the adjacent faces making equal angles (45 degrees) with H, and a face making an angle of 30 degrees with V.

B_h is at (2″, 4.5″), and B_v at (2″, 0.75″).

The T plane coincides with the bottom edge of the cube, and the R plane passes through the corner E which is farthest from V.

The line of shade is B A D C E G B; the projection of the part of its shadow shown on the T plane is $B_h\ A_1\ D_1\ F_1\ G_1\ B_h$, and of that part shown on the R plane $D_2\ C_2\ E_v\ F_2\ D_2$. G F is the only part of the edge G E which casts a shadow on the T plane, and E F the part which casts a shadow on the R plane.

The faces that are in the shade may be found by the method of Art. 34. A B D is the only visible face of the cube in the shade.

(*b*) Find the projections of the shaded part of a cylinder, and of the shadow of the cylinder on a horizontal plane (T) coinciding with the lower base, and on a vertical plane (R) parallel to V whose horizontal trace is at $y = 5.75''$. Show the projection of the entire shadow on the T plane; on the R plane show only that part which is above the T plane.

While it is conventional to regard the projections of rays of light on H and V as making an angle of 45 degrees each with a horizontal, nevertheless it is optional with the draftsman to assume them in any reasonable direction. In this problem let the projections of rays on H make angles of 30 degrees, and those on V an angle of 45 degrees, with the horizontal.

The axis of the cylinder is taken perpendicular to H and parallel to V.

C_h is at (8″, 4.5″), and C_v at (8″, 3.25″).

The line of shade is D′ D G E E′ K′ D′; the projection of its shadow on the T plane is $D_h\ D_1\ F_1\ E_1\ E_h\ K_h\ D_h$, and the part of the shadow shown on the R plane $F_2\ G_2\ E_2\ N_2\ F_2$. E N is the only part of the line of shade E E′ whose shadow shows on the R plane.

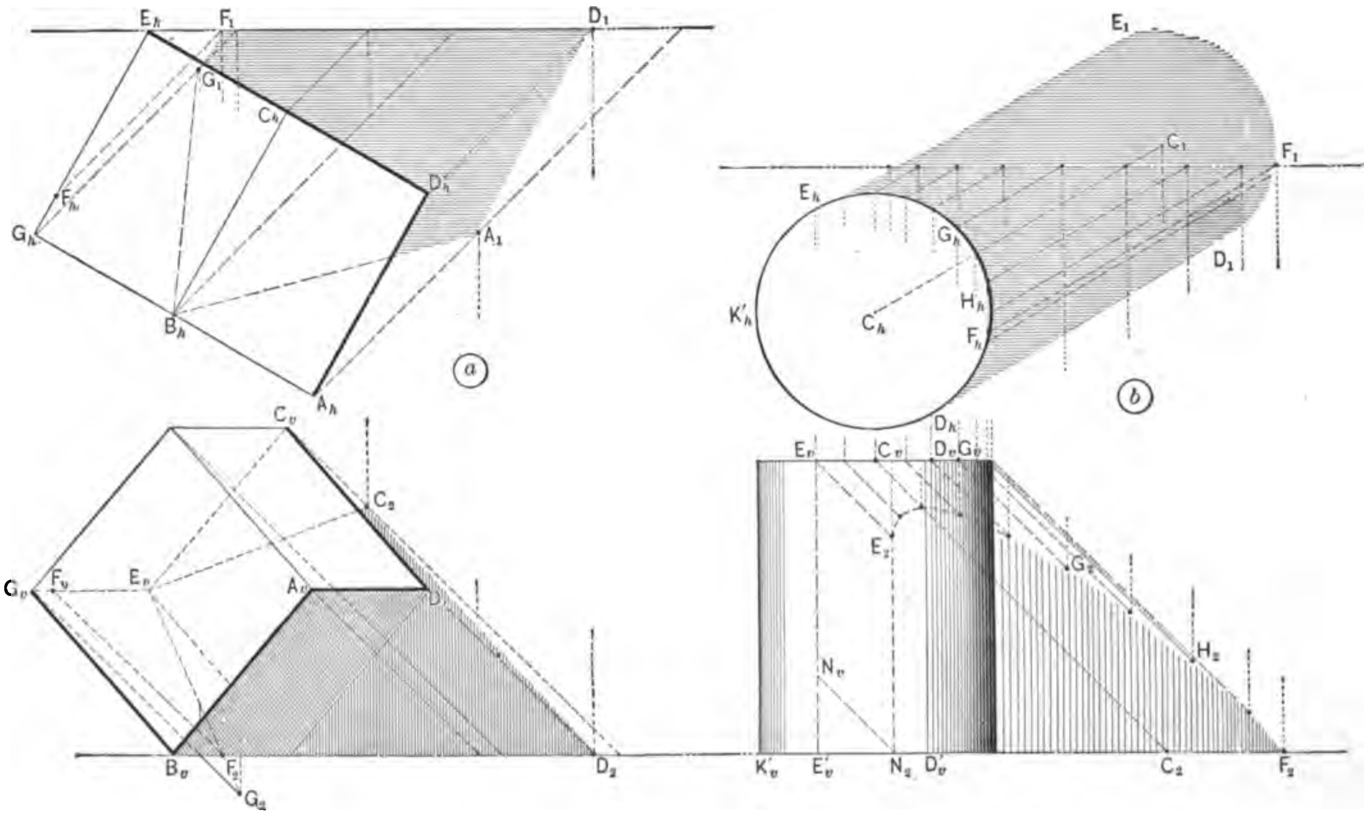
The projection on the T plane of the outline of the shadow $D_1\ F_1\ E_1$ is a semicircle with its center at C_1, equal to the semicircle D G E which casts the shadow. The projection on the R plane of the shadow of the same semicircle is found by determining the projections of the shadows of its several points and joining them by an irregular curve.

The shaded part of the cylinder is the one-half which includes the bottom base and that part of the lateral surface away from the source of light bounded by the line of shade, viz., D D′ F′ G′ E′ E G F D.

A few parallel shade-lines, gradually increasing in their distance apart, are drawn at the left of the figure to give the appearance of a curved surface, notwithstanding that part of the surface is wholly in the light.

Note: The application of the principles of light and shade as they apply to the representation of curved surfaces will not be attempted here; should the student desire to look farther into the subject he is referred to Randall's Shades, Shadows, and Perspective; J. E. Hill's Shades, Shadows, and Perspective; and Notes on Shades, Shadows, and Perspective, by C. E. Crandall, revised and enlarged by Walter L. Webb.

SHADES AND SHADOWS



63. **Perspective.**—In the projections which have preceded, the projecting lines have been parallel to each other, that is, the position of the eye has been assumed as at an *infinite distance from the plane of projection.* If the position of the eye is at a *finite distance from the plane of projection,* the projecting lines *will not be parallel,* but will meet the plane of projection at different angles. The points where the projecting lines meet the plane of projection determine the projection as in preceding cases. *Such a projection is called a perspective.*

A perspective may be imagined if one looks with one eye only at an object, such as a house, through a window, and sees traced on the glass the outlines of the house as they are projected upon it.

A perspective is a true representation of an object as the eye sees it, but it is useless as a working drawing, *because the projections of parallel and equal lines have not the same lengths except when the lines they represent are both parallel to the plane of projection and at the same distance from it.*

A photograph is a perspective, and since an existing object or body may be photographed much more cheaply and satisfactorily (except for very simple objects) than a perspective drawing can be made, it is evident a perspective serves the purpose of representing non-existing bodies, e.g., a dwelling about to be erected.

To make a mathematical or accurate perspective of a body which does not exist, it is first necessary to draw orthographic projections on H and V, and locate the orthographic projections of the position of the eye. Also it is necessary to deal with the orthographic projections of the projecting lines rather than with the lines themselves.

The projecting lines are called *visual rays.*

The plane of projection is called the *picture-plane.*

The position of the eye is called the *point of sight,* and the orthographic projection of the point of sight on the picture-plane is called the *center of the picture.*

The trace on the picture-plane of a horizontal plane at the height of the eye is called the *horizon. The center of the picture is always on the horizon.*

The perspective of a point is determined directly by finding where the projecting line of that point meets the picture-plane, or otherwise by finding the intersection of the perspectives of two lines which pass through the point. The last method is the one often adopted, because it does not require the vertical projection of the body to be drawn, but uses a vertical line on which are marked off the heights of the different points. This line is called the *line of heights.*

The projections of all lines which are not parallel to the picture-plane, but which are parallel to each other, have one point of perspective in common which is called the *vanishing-point.*

The vanishing-point of a system of parallel lines is determined by finding where a line through the point of sight

parallel to the lines of the system pierces the picture-plane. The proof of this is as follows: Any one of the parallel lines of the system and the line through the point of sight parallel to it form a plane which intersects the picture-plane in a line which contains the perspective of the one line. Any other one of the parallel lines and the same line through the point of sight form another plane the trace of which on the picture-plane contains the perspective of the second line. Since both these planes contain the parallel line through the point of sight in common, their traces on the picture-plane will intersect in the point where that line pierces the picture-plane; hence this point will be common to the perspectives of the two parallel lines and all other lines parallel to them.

The center of the picture is the vanishing-point of all lines perpendicular to the plane of the picture. Such lines are called *perpendiculars.*

The horizon will contain the vanishing-points of all lines that are horizontal whatever angle they may make with the picture-plane. The vanishing-point of horizontal lines which make the angle of 45 degrees with the picture-plane will be in the horizon at a distance from the center of the picture, equal to the distance of the point of sight from the picture-plane. The vanishing-point of such lines is called the *point of distance*, and the lines are called *diagonals.*

The perspectives of all lines parallel to the picture-plane are parallel to the projections of the lines. The vanishing-point of a system of such lines is at infinity, that is, their perspectives are parallel to each other. All lines parallel to the picture-plane which are vertical are called *verticals*, and if horizontal they are called *horizontals.*

The two lines which are used to determine the perspective of a point are in general a *perpendicular* and a *diagonal*, or a *vertical with either of these two*, or a *horizontal with either of these three.*

64. *To find the perspectives of shadows on any surface in general, find the perspective of the point where the ray of light through the point pierces the surface.* If it is a shadow on a horizontal plane, find the perspective of the ray of light and the perspective of the projection of that ray on the plane, and where these perspectives meet will be the perspective of the shadow of the point.

If the rays of light have the direction as given in Art. 34, then the projection of rays on H being diagonals, their perspectives will vanish in the horizon at the point of distance (D_v), and the vanishing-point of the perspective of rays will vanish at a point in a vertical line through D_v as far below it as D_v is from the center of the picture (S_v). But the direction of the rays of light may be assumed at will so long as the conditions are reasonable; therefore R_h, somewhere in the horizon, may be taken as the vanishing-point of the perspectives of the projections of rays on a horizontal plane, and R_v somewhere directly below as the vanishing-point of the perspective of the rays. The nearer to the horizon R_v is taken the more nearly horizontal will be the rays of light, and the nearer R_h is to S_v the more nearly are the rays of light behind the draftsman.

65. PLATE 35.

Find the perspective of a cube, the perspective of its shade, and the perspective of its shadow on a horizontal plane. The edge of the cube is 2″, and faces are parallel to H and V.

A_h is at (3.5″, 5″), and A_v at (3.5″, 3.5″).

K_h L_h is the trace on H of the picture-plane at $y=4.75''$.

S_v D_v is the horizon at $y=4.25''$.

A_v' B_v', the trace on the picture-plane of the horizontal plane (T) upon which the cube rests, is at $y=1.5''$.

S_v is the center of the picture at $x=8.25''$.

D_v is the distance-point (vanishing-point of diagonals) at $x=12''$.

R_h is the vanishing-point of the projection of rays of light on the T plane at $x=10''$, and R_v the vanishing-point of rays of light at $y=0.75''$.

The picture-plane and V are one and the same plane.

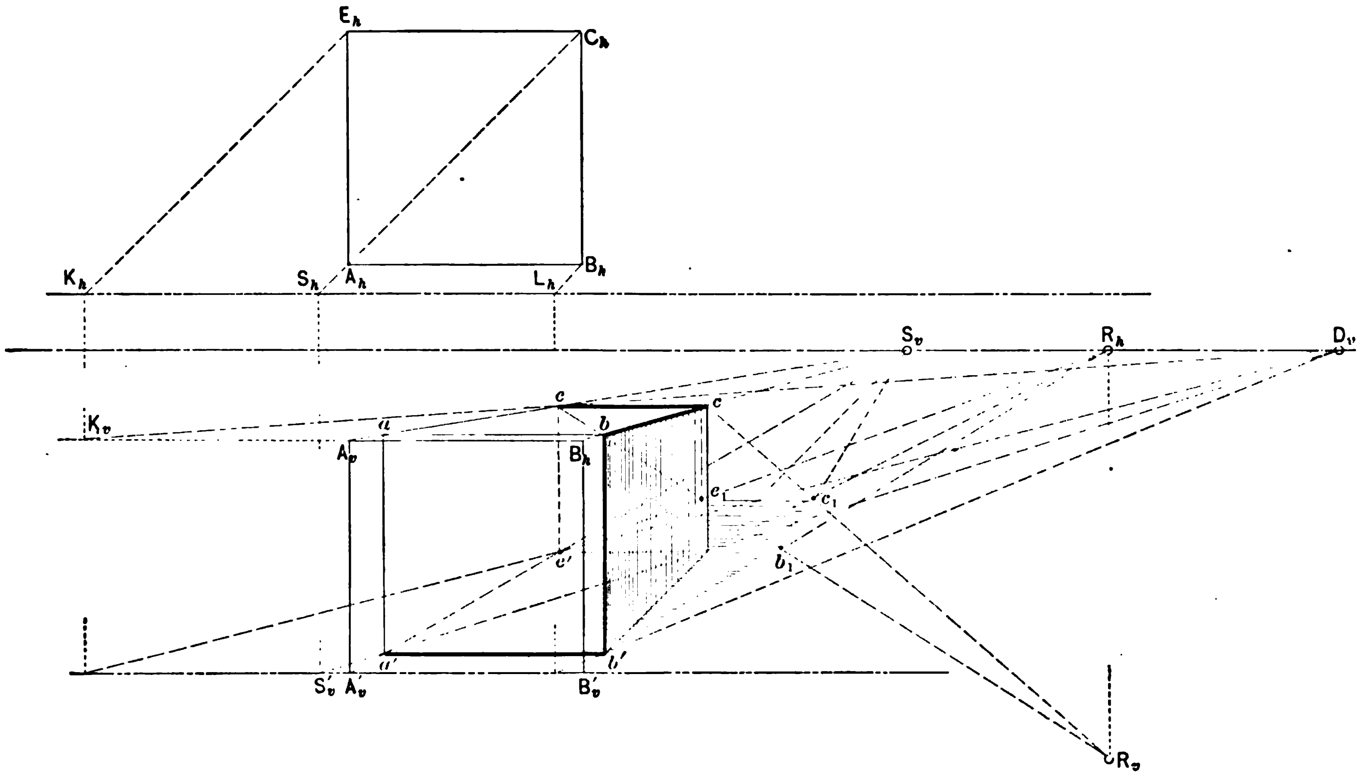
Find the perspective of the corner A′ by a diagonal and a perpendicular. S_v' D_v is the perspective of the diagonal, and A_v' S_v the perspective of the perpendicular. The two perspectives meet at a', which is the perspective of the corner A. For the diagonal through A is projected on H in the line A_h S_h, and on V in the line A_v' S_v'. This diagonal meets the picture-plane at a point whose projection on H is S_h, and on V at S_v'; therefore S_v' is the perspective of one point of the diagonal, and the vanishing-point of diagonals D_v is another point.

Find the perspective of the corner A by a vertical and a perpendicular. a' a is the perspective of the vertical through A, and A_v S_v the perspective of the perpendicular. For the perspective of the vertical a' a is parallel to the projection of the vertical on the picture-plane; also the perspective of the perpendicular will vanish at the center of the picture, and another point of its perspective will be where the perpendicular meets the picture-plane.

Find the perspective of the corner B by a horizontal and a perpendicular, of B′ by a horizontal and a vertical, of C′ by a diagonal and a perpendicular, of C by a vertical and a perpendicular, and of E′ by a perpendicular and a diagonal. The perspective of E may be found by the perspective of the horizontal c e, and that of the perpendicular a e, but a better way would be to determine it by the perspectives of the vertical e' e and the horizontal c e, because the intersection of the last two lines meeting more nearly at right angles gives a more accurate result. Avoid determining the perspectives of points by lines which make a small angle at their intersection.

To find the perspective of the shadow of the cube: The perspective of the line of shade is b' b c e e'; this line casts the perspective of the outline of the shadow. The perspective of the shadow of b' is itself, since it is in the plane of the shadow. To determine the perspective of the shadow of b, draw b' R_h, the perspective of the projection on the T plane of the ray of light through B, and b R_v, the perspective of the ray of light through B; where these two lines meet will be the perspective of the shadow of B on the T plane. The line b c is the perspective of a perpendicular which is parallel to the T plane, hence its shadow on the T plane will be a perpendicular, and the perspective of its shadow will vanish at S_v. b_1 S_v is the perspective of the indefinite shadow of b c, and where the line c R_v intersects this line is the perspective of the shadow of the corner C on the T plane. c_1 e_1, parallel to c e, is the perspective of the shadow of the edge C E, and e_1 e' that of the edge E E′.

PERSPECTIVE



66. PLATE 36.

Find the perspective of a prism, the perspective of its shade, and the perspective of its shadow on the horizontal plane (T) upon which it rests. The height is 3″, the base rectangular 2″ by 1.5″, with a wider face, making an angle of 30 degrees with V and the bases parallel to H.

B_h is at (4.25″, 6″) and B_v at (4.25″, 4″).

B_v B_v', the projection on V of the edge B B′, may be taken as the line of heights.

The trace on H of the picture-plane is at $y=4.5''$,

The horizon at $y=4.25''$,

S_v at $x=6.25''$,

D_v at $x=12''$,

R_h at $x=11''$, and

R_v at $y=1.5''$.

P_v is the vanishing-point of the perspectives of the edge B C and all edges parallel to it. It is found thus: Produce B_h C_h until it meets the trace of the picture-plane, then $m\,n : m\,k = S_v\,D_v : S_v\,P_v$. The vanishing-point of the perspectives of the edge A B and all others parallel to it might be found in a similar way, but since the point falls outside the limits of the drawing it is not used.

Find the perspectives a' and b' by perpendiculars and diagonals, draw a' P_v and the point of intersection with the line E_v' S_v, the perspective of a perpendicular through E′, will be the perspective of E′. b is the intersection of b' b and B_v S_v, the perspectives of a vertical and a perpendicular respectively through B. $b\,c$ vanishes at P_v. a is the intersection of a' a and h D_v, the perspectives of a vertical and a diagonal respectively through A.

The only face in view which is in the shade is $a'\,a\,e\,e'$.

The perspective of the shadow of the edge A A′ on the T plane is a' a_1, vanishing at R_h. a_1 e_1, the perspective of the shadow of A E, vanishes at P_v. c' c_1 vanishes at R_h, and the point where the line c R_v meets the line c' R_h is the perspective of the shadow of the corner C.

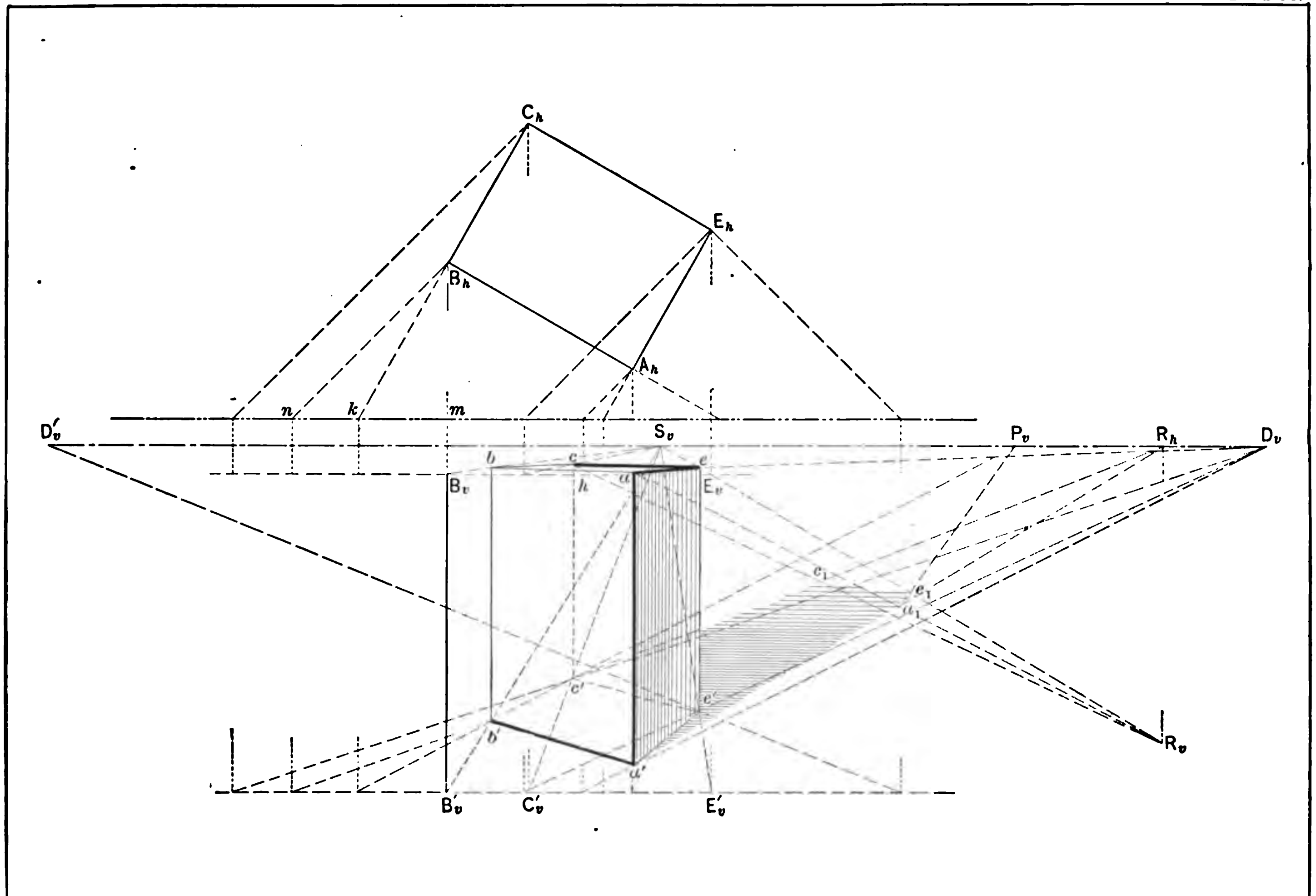

C_h
E_h
B_h
A_h
n
k
m
D'_v
S_v
P_v
R_h
D_v
b
c
e
B_v
h
a
E_v
c_1
e_1
a_1
c'
e'
b'
a'
R_v
B'_v
C'_v
E'_v

PLATE 37.

67. Find the perspective of the frustum of a cylinder as given in Plate 24 (*c*), and of its shadow on a horizontal plane (T) upon which it rests. The projection on H of the center of the lower base is at (5″, 6″), and the projection on V of the same point at (5″, 1.5″).

The picture-plane and V are one and the same plane.

The trace on H of the picture-plane is at $y=5''$.

The horizon at $y=4.25''$.

S_v is at $x=7.25''$.

D_v' at $x=2.5''$,

R_h at $x=9''$, and

R_v at $y=2.5''$.

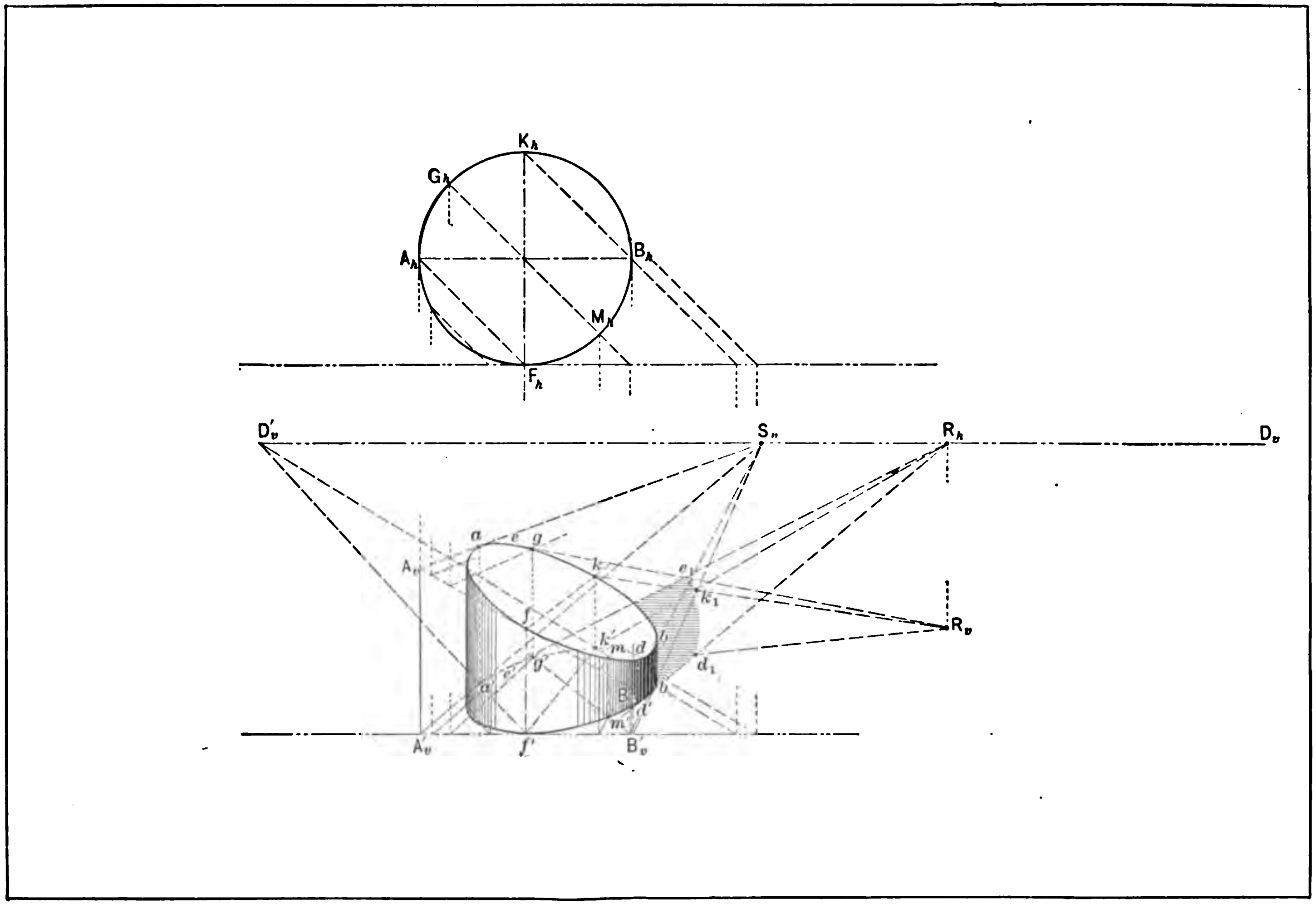
A_v B_v is the projection on V of the upper base.

Find a' by a perpendicular and a diagonal through A, and a by a vertical and a perpendicular through A.

Find the perspective of other points in the boundary of the lower base by perpendiculars and diagonals, and the perspective of corresponding points immediately above by verticals and perpendiculars. The perspective of each perpendicular has a point on the line A_v B_v, which is the projection of the point on V, and it vanishes at S_v. Find a sufficient number of points in the bases to accurately outline their perspectives by joining them with an irregular curve.

To find the perspective of the shadow on the T plane draw lines through R_h tangent to the lower base at d' and e', and through R_v tangent to the upper base at e and d. d d' and e e' are the perspectives of the lines of shade on the surface of the cylinder, and e' R_h and d' R_h are the indefinite perspectives of these lines of shade on the T plane respectively, and where e R_v meets e' R_h is the limit on one side and similarly d_1 is the limit on the other. Through the perspective of points on the lower base draw lines to R_h, and from the corresponding points on the upper base draw lines to R_v; where these lines meet will be the perspectives respectively of the shadows of the points on the T plane. For example, k_1 is the perspective of the shadow of k, k' R_h meeting k R_v at k_1. In this way the line e_1 k_1 d_1 is determined.

Without discussing the reasons for the effect of light on a curved surface as seen by the eye, it is sufficient to say in this connection that a surface in the shade will appear to grow lighter, and conversely a surface in the light will appear to grow dearkr as the surfaces recede. In shading the perspective of the surface of the cylinder students should observe these rules.



Kh
Gh
Ah
Bh
Mh
Fh
D'v
Sv
Rh
Dv
a
e
g
Av
k
e1
k1
f
k'
m
d
b
d1
Rv
a'
e'
g'
B
b'
d'
m'
A'v
f'
B'v

PLATE 38.

68. Find the perspective of the cube with a section cut from one face and having a square hole running through it, as shown in Plate 13 (*b*).

B_h is at (3.25″, 5.75″) and B_v at (3.25″, $2\frac{5}{8}$″).

The plane of the section is taken parallel to the picture-plane.

The trace of the picture-plane on H is at $y=5''$.

The horizon is at $y=4.25''$.

S_v is at $x=6.25''$,

D_v at $x=12''$,

R_h at $x=10''$, and

R_v at $y=0.5''$.

D_v and D_v' are at equal distances from S_v.

P_v is the vanishing-point of the perspective of the edge B N, and all edges parallel to it, found as explained in Art. 66.

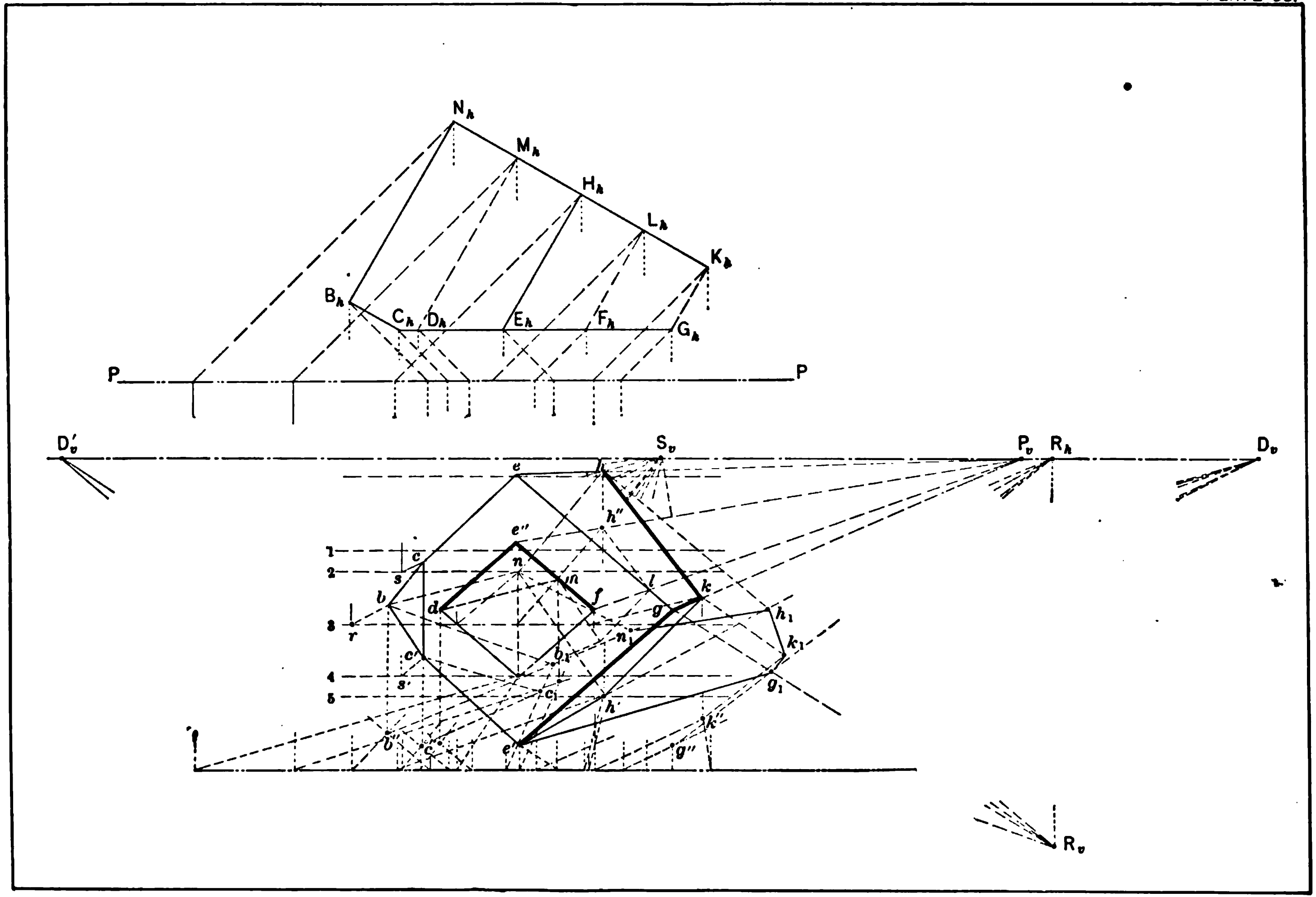
The horizontal lines 0, 1, 2, etc., are at heights above the T plane equal to the heights of the edges E H, D M, the point C, etc., respectively.

In finding the perspectives of the different corners of the cube and the hole, the following method is observed: Find the perspective of a point on the T plane, directly under the corner, by a perpendicular and a diagonal. This is the perspective of the projection of the corner on the T plane. A vertical line is drawn through this point and is the perspective of a vertical through the corner. The point where the perspective of a perpendicular, a horizontal, or a diagonal through the corner meets this vertical line is the perspective of the corner. This method serves to aid in finding the perspective of the shadow, for the perspective of the projection of the corner on the T plane is one point of the perspective of the projection of the ray of light which passes through the corner.

b'' is the perspective of the projection of B on the T plane, $b''\,b$ the perspective of the vertical through B, and $r\,b$, vanishing at S_v, the perspecttve of the perpendicular through B, giving b the perspective of the corner B. c'' is the perspective of the projection of the corners C and C′ on the T plane, $c''\,c$ the perspective of the vertical, and $s\,c$ and $s'\,c'$ the perspectives of the perpendiculars through C and C′ respectively. The other corners are found in a similar manner.

The perspective of the line of shade is $e'\,g\,k\,h\,n\,b\,c\,e'$, and the perspective of its shadow on the T plane $e'\,g_1\,k_1\,h_1\,n_1\,b_1\,c_1'\,e'$.

To find the perspective of the shadow of the corner g, for example, draw from g'', the perspective of the projection of the corner G, a line to R_h, the vanishing-point of the projection of rays of light on the T plane, and a second line from g to R_v; the point where these lines intersect, is the perspective of the shadow of g on the T plane. Again, draw $k''\,R_h$ and $k\,R_v$, and the point where these lines intersect gives k_1. Similarly all the other points of the outline of the perspective of the shadow may be found. The face $e'\,g\,k\,h'$ is the only face in the shade which is in view.



Nh
Mh
Hh
Lh
Kh
Bh
Ch
Dh
Eh
Fh
Gh
P
P
D′v
Sv
Pv
Rh
Dv
Rv
e
e″
h″
n
c
s
b
d
f
l
k
g
h1
k1
r
n1
c′
b1
s′
c1
g1
h′
k′
b″
c″
g″
e′
1
2
3
4
5

PLATE 39.

(Continuation of the shadow of Plate 38.)

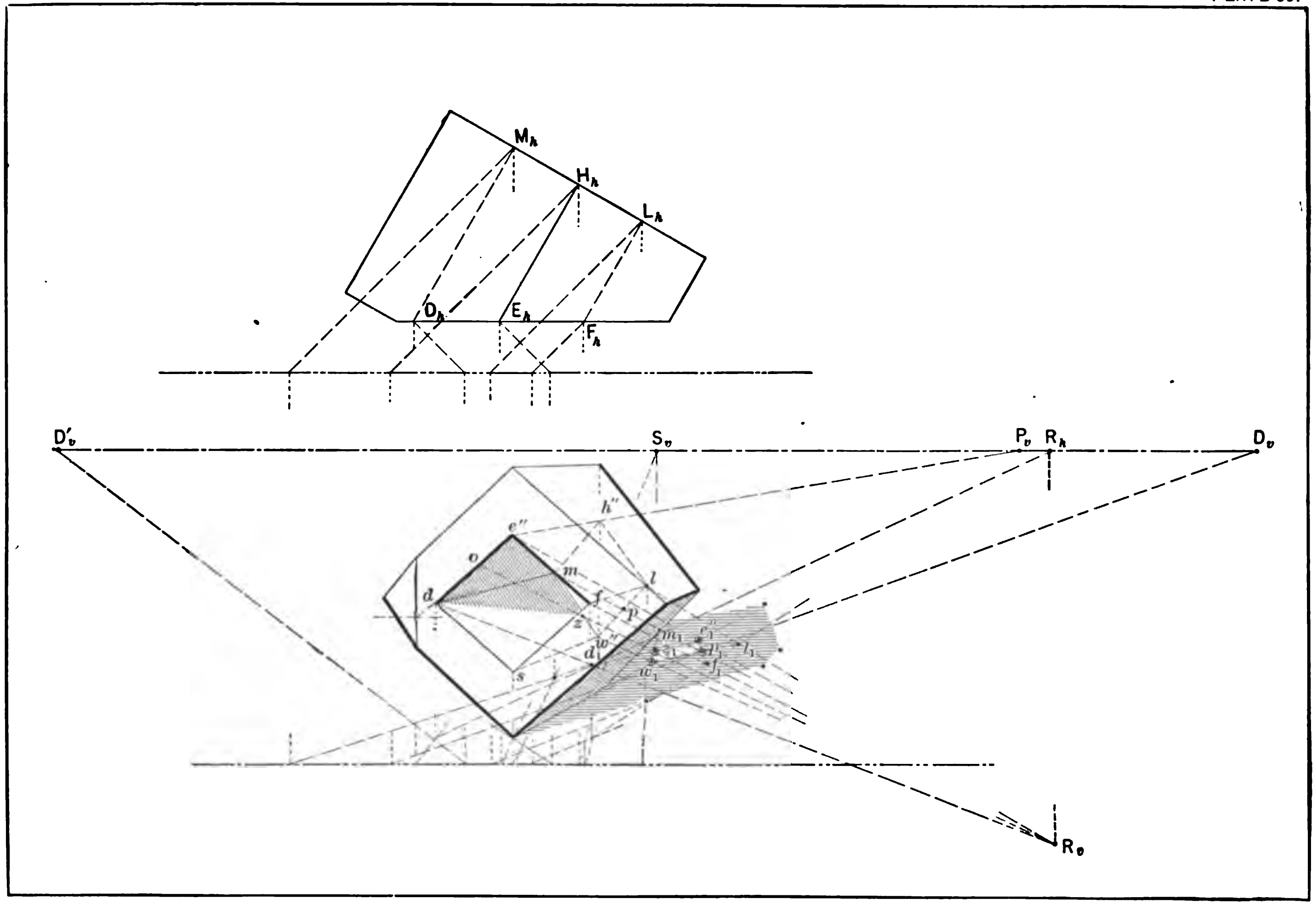
Find the perspective of the shadow of the edge of the hole, $d\,e''$, on the face of the hole $d\,m\,w''s$, and determine the outline of the bright spot in the shadow on the T plane caused by the light shining through the hole.

The face of the hole $d\,e''\,h''\,m$ is in the shade, this being the only face in view that is in the shade.

Find the perspectives of the shadows on the T plane of all the edges of the hole, and the outline of the bright spot will thus be determined. If, however, it is clear to the student which edges cast the shadows which are the outlines of the bright spot, it will be necessary to find the perspectives of the shadows of those edges only.

Find the perspectives of the shadows $m\,w''$, $w''\,l$, $e''\,f$, and $d\,e''$ on the T plane in the usual way. These are the edges, parts of which will cast the shadows which are the outlines of the bright spot in the shadow on the T plane.

At z_1, where $d_1\,e''$ meets $m_1\,w_1$, draw a line to R_v and extend it backwards to o. This is the perspective of the ray of light which limits the shadow of $d\,e''$ on the T plane. $o\,e''$ casts the shadow $z_1\,e_1''$ on the T plane, and $d\,o$ the shadow $d\,z$ on the face $d\,m\,w''s$.


M_h
H_h
L_h
D_h
E_h
F_h
D'_v
S_v
P_v
R_h
D_v
R_v
h''
e''
o
m
l
d
f
p
z
w''
d_1
s
m_1
e_1
h_1
l_1
f_1
w_1

PART II.

PROBLEMS IN DESCRIPTIVE GEOMETRY.

GENERAL INSTRUCTIONS AND CONVENTIONS.

1. **Instructions.**—The *planes of projection* and *auxiliary planes* are assumed to be *transparent*, and *given* and *required planes opaque*.

If a *given line is in view*, its projections are to be drawn *medium, full, black;* and if *hidden, light, broken, black*.

If a *required line is in view*, its projections are to be drawn *heavy, full, black*, and if *hidden, light, broken, black*.

If the *trace of a given plane is in view*, it is to be drawn *medium, full, black;* and if *hidden, light, broken, black*.

Construction lines are to be drawn *light, full, red; lines joining two projections of the same point, light, dotted, red;* and *dimension-lines*, when used, are *light, full, red*, but the arrow-points marking the limits and the numerals are black.

Traces of auxiliary planes (planes used for construction) are to be *light, dash-and-two-dot, red*.

Given points are to be enclosed by *very small circles in black*.

The foregoing rules do not apply to the projections of the outlines of surfaces.

For the size of the plates and their subdivision, and for explanation of coordinate axes and ordinates, see Part I, Art. 19.

2. **Conventions.**—The four dihedral angles formed by the intersection of the H and V planes will be designated as follows:

The first angle, above H and in front of V;

The second angle, above H and behind V;

The third angle, below H and behind V; and

The fourth angle, below H and in front of V.

The capitals H, V, P, and S will designate the *horizontal, vertical, profile*, and *supplementary planes of projection*, respectively.

An *auxiliary plane* will be designated by either R, T, or U.

The *ground-line* will be called H V between the *horizontal* and *vertical* planes, between the *horizontal* and *profile* planes H P, and between the *vertical* and *profile* planes V P.

A *point in space* will be designated by a capital letter, e.g. P, and *its projections* on H, V, P, and S, by P_h, P_v, P_p, and P_s, respectively.

A *line in space* will be designated by referring to any two points in the line, as the line M N, and *its projections* to the corresponding projections of those points.

The problems from 1 to 18 are to be constructed and numbered consecutively in the squares of their respective plates (see Part I, Diagram A). The ground-line, H V, will bisect the square in each of the problems.

3. Problems.

PLATE I.

Prob. 1. Represent four points A, B, C, and D:

A in the first angle $1\tfrac{3}{16}''$ from H, $\tfrac{13}{16}''$ from V, $x=\tfrac{13}{16}''$, *

B in the second angle $1\tfrac{3}{16}''$ from H, $\tfrac{13}{16}''$ from V, $x=1\tfrac{3}{8}''$;

C in the third angle $1\tfrac{3}{16}''$ from H, $\tfrac{13}{16}''$ from V, $x=2\tfrac{1}{8}''$;

D in the fourth angle $1\tfrac{3}{16}''$ from H, $\tfrac{13}{16}''$ from V, $x=2\tfrac{13}{16}''$.

Prob. 2. Represent two right lines MN and OP:

MN in the third angle, perpendicular to H, $1''$ from V, $1\tfrac{3}{8}''$ long;

M lying in H, $x=\tfrac{13}{16}''$;

OP in the third angle parallel to V and $\tfrac{13}{16}''$ from V;

O, $\tfrac{3}{8}''$ from H, $x=1\tfrac{1}{4}''$;

P, $1\tfrac{1}{4}''$ from H, $x=3''$.

Prob. 3. Represent the plane of a line MN and a point P:

MN in the third angle, parallel to AB, $1\tfrac{1}{4}''$ from H, $\tfrac{7}{8}''$ from V;

P in the third angle, $1''$ from H, $\tfrac{3}{4}''$ from V, $x=1\tfrac{1}{2}''$.

Prob. 4. Represent a plane, R, in the third angle passing through three points A, B, and C:

A in the first angle, $1''$ from H, $1\tfrac{1}{4}''$ from V, $x=\tfrac{1}{2}''$;

B in the third angle, $\tfrac{1}{4}''$ from H, $\tfrac{7}{8}''$ from V, $x=1\tfrac{1}{2}''$;

C in the fourth angle, $1\tfrac{1}{8}''$ from H, $\tfrac{3}{8}''$ from V, $x=3''$.

Prob. 5. Represent a plane R in the third angle passing through a point P and perpendicular to a line MN:

P in the third angle, $1''$ from H, $\tfrac{7}{8}''$ from V, $x=2\tfrac{3}{4}''$;

M in the third angle, $1\tfrac{1}{4}''$ from H, $\tfrac{1}{8}''$ from V, $x=\tfrac{5}{8}''$;

N in the second angle, $\tfrac{3}{4}''$ from H, $1''$ from V, $x=1\tfrac{1}{2}''$.

Prob. 6. Given the points and plane of Problem 5, to represent the point where MN pierces R at G, and also show the true length of the line joining P with G.

(x represents the distance of the projections of the point from the vertical axis of the square.)

4.

PLATE II.

Prob. 7. Represent in the third angle the intersection of two planes, R and T.

The traces of the R plane meet at $x=\frac{1}{2}''$.

The traces of the T plane meet at $x=2\frac{3}{4}''$.

The H trace of the R plane makes with HV an angle of 75°.

The V trace of the R plane makes with HV an angle of 60°.

The H trace of the T plane makes with HV an angle of 120°.

The V trace of the T plane makes with HV an angle of 105°.

Prob. 8. Show the true size of the angle represented between the lines MK and OK.

MK is in the third angle parallel to HV, 1″ from H, ½″ from V.

O K is in the third angle.

O is 1¼″ from H, 1½″ from V, $x=2\frac{1}{2}''$.

For K, $x=1\frac{1}{4}''$.

Prob. 9. Represent on a profile plane, through the point O, the solution of Prob. 8, looking in the direction from right to left.

For the V P trace, $x=\frac{3}{4}''$.

Prob. 10. Represent the intersection, MN, of two planes parallel to HV lying across the third angle.

The T plane makes an angle of 30° with H and 60° with V; the H trace is 1″ from HV.

The R plane makes an angle of 75° with H and 15° with V; the H trace is ⅜″ from HV.

Prob. 11. Represent the point where a line, M N, pierces a plane, T, and the projection of the line on the plane.

MN is in the third angle, parallel to HV, 1″ from H, ½″ from V.

The H trace of the plane makes an angle of 30° with HV, above.

The V trace of the plane makes an angle of 60° with HV, below.

The traces intersect at $x=\frac{1}{2}''$.

Prob. 12. Represent the point where a line, MN, pierces a plane, T.

MN is in the third angle.

M is 1″ from H, ½″ from V, $x=1\frac{1}{4}''$.

N is ½″ from H, 1″ from V, $x=1\frac{1}{4}''$.

The plane is taken as in Prob. 11.

5. PLATE III.

Prob. 13. Trisect the angle between two right lines, MN and MP, and show the projections of the trisecting lines.

MN is in the third angle, parallel to H, $\frac{1}{4}''$ from H.

M is $1\frac{1}{4}''$ from V, $x=\frac{3}{4}''$.

N is $\frac{1}{4}''$ from V, $x=2\frac{3}{4}''$.

P is in the third angle, $1''$ from H, $\frac{1}{4}''$ from V, $x=2''$.

(Suggestion: Revolve the plane of the two lines about MN, as an axis, until it is parallel to H.

Prob. 14. Represent a plane passing through a point Q and parallel to the lines MN and OP.

Q is in the third angle, $\frac{1}{4}''$ from H, $\frac{3}{4}''$ from V, $x=\frac{5}{8}''$.

MN is in the third angle, parallel to HV, $\frac{1}{2}''$ from H, $\frac{1}{2}''$ from V.

O is in the third angle, $\frac{1}{4}''$ from H, $\frac{3}{4}''$ from V, $x=1''$.

P is in the third angle, $1\frac{1}{8}''$ from H, $\frac{1}{8}''$ from V, $x=2\frac{1}{2}''$.

Prob. 15. Find the true distance from a point P to a line MN.

P is in the third angle, $1''$ from H, $\frac{1}{2}''$ from V, $x=1''$.

M is in the third angle, $1''$ from H, $1\frac{1}{4}''$ from V, $x=\frac{3}{4}''$.

N is in the third angle, $1''$ from H, $\frac{1}{8}''$ from V, $x=2\frac{7}{8}''$.

Prob. 16. Find the common perpendicular, X Y, between the lines MN and OP.

MN is in the third angle, perpendicular to H, $1''$ from V, $x=\frac{3}{4}''$.

O is in the third angle, $1''$ from H, $\frac{1}{4}''$ from V, $x=1\frac{1}{4}''$.

P is in the third angle, $\frac{1}{8}''$ from H, $1\frac{1}{8}''$ from V, $x=2\frac{7}{8}''$.

Prob. 17. Represent an oblique plane, T, not parallel to HV, making an angle of 60° with H and 45° with V, and passing through a point P.

P is in the third angle, $\frac{1}{2}''$ from H, $\frac{1}{2}''$ from V, $x=3''$.

Prob. 18. Represent a line making an angle of 30° with H and 45° with V, and passing through a point P.

P is in the third angle, $1''$ from H, $\frac{1}{2}''$ from V, $x=2\frac{1}{2}''$.

6. Shade-lines.— The projection of surfaces in the problems to follow will not be sectioned-lined to represent shaded parts, but shade-lines will indicate which of the surfaces in view are in the shade. *Shade-lines* are those representing edges which join surfaces, or faces, one in the light and the other in the shade.

In *curved surfaces*, such as the cylinder and the cone, the elements of shade do not coincide with the elements of outline; the side outlines will have, therefore, no distinction made in their grade. The outline of the bases should conform to the rule which will be given later.

The H and V projections of rays of light will each make an *angle of 45 degrees with* H V, because the rays of light are assumed to approach the H and V planes from the first angle, in the direction of the body diagonal of a cube whose faces are parallel to H and V.

When a body is made up of plane surfaces, or faces, in order to ascertain whether a face is in the light or in the shade, it is necessary to determine whether rays of light pierce the surface uninterruptedly, or are intercepted by another face. For example, in Fig. 8, Part I, to determine whether the face A B E F is in the shade, draw the trace on H of a plane of rays, R_h, perpendicular to H, intersecting the faces A′ B′ E F and A B E F in the lines M N and N H respectively. If any ray of light in this plane can be drawn which will pierce the face A B E F before it pierces the face A′ B′ E F, then the face A B E F is not in the shade; otherwise it is in the shade because the other face intercepts the rays of light. The ray R drawn through any point of the line M N, as G, in the face A′ B′ E F, pierces the other face at H, and all other rays of light piercing the face A′ B′ E F between G and N will also pierce the face A B E F between N and H. Therefore the face A B E F is in the shade, because the face A′ B′ E F intercepts the rays of light in their path towards the first-named face.

Again, the plane of rays of which R_h is the trace on H cuts from the face A′ B′ C′ D′ the line M P, and from the face C′ D′ K L the line P Q. The ray R′, lying in this plane and meeting the first-named surface in W, does not intersect the line P Q, which also lies in the same plane of rays and in the face C′ D′ K L. No ray in that plane drawn through any point in the line M P will intersect the line P Q. The face C′ D′ K L is not in the shade by reason of the interception of rays by the face A′ B′ C′ D′.

Having determined which faces of a body are in the light and which are in the shade, the following rule is to be observed: *Those lines of a drawing representing edges between two faces, both in the light, and those representing edges between two faces, both in the shade, are to be drawn light or medium; and lines representing edges between two faces, one in the light and the other in the shade, are to be drawn heavier. However, lines which represent hidden edges are to be drawn light, broken, whatever the conditions of light and shade.*

In *profile* and *supplementary projections* it will be assumed that the *object* represented *remains stationary*, while the *planes of projection change* their positions; *hence the shade-lines are the same lines in these projections as in the projections on* H *and* V.

The direction of the rays of light as given above is that usually adopted, but there is no good reason other than this why the direction may not be assumed at the will of the draftsman. It is the custom of some draftsmen to disregard any reference to light and shaded faces, but to arrange the light and heavy lines to produce a pleasing effect in the drawing. The student is recommended to observe exact methods until experience teaches him to readily assume conditions of light and shade which do not produce glaring inconsistencies.

7. PLATE IV.

Prob. 19. Represent a plane tangent to a prolate spheroid in the third angle, at a point, P, on its surface.

The transverse axis is 3″ long perpendicular to H, $1\frac{3}{4}$″ from V; $x=5\frac{1}{2}$″; the highest point, M, is $\frac{1}{4}$″ from H.

The conjugate axis is $2\frac{1}{4}$″ long.

The point P is on the upper surface; for its H projection $x=6$″, $y=6\frac{1}{2}$″.

Note: *Take a whole plate for this problem and those to follow and let the origin be at the lower left corner of the border-line of the plate, and let* H V *divide the plate in halves.*

8. PLATE V.

Prob. 20. Find the intersection of a right cone with a plane.

Show the section in its true size, make a profile of the lower part and develop it.

The cone is in the third angle; the axis vertical, 3″ long, 2″ from V at $x=6\frac{1}{4}$″; the vertex is in H, and the diameter of the base is 3″.

The plane is perpendicular to V and cuts the axis at a point $1\frac{1}{2}$″ from the vertex; the V trace of the plane inclines downwards to the right, making an angle of 45 degrees with HV.

Let the axis of the cone in profile be at $x=10$″.

Let the transverse axis of the true size of the section be parallel to H V at $x=10$″, $y=6\frac{1}{4}$″ for the middle point.

In the development, or pattern, let the longest element coincide with a horizontal line at $y=4\frac{1}{4}$″ and the vertex be at $x=4$″, $y=4\frac{1}{4}$″.

9. PLATE VI.

Prob. 21. The same as Prob. 20, except that the cutting plane is to be taken so as to cut a section from the right cone which shall be a parabola.

10. PLATE VII.

Prob. 22. The same as Prob. 20, except that the cutting plane shall cut a section which shall be an hyperbola, the V trace making an angle of 75 degrees with H V.

11. PLATE VIII.

Draw an oblique cylinder, the elements making an angle of 30 degrees with H and 45 degrees with V, inclining downward to the right and toward V. The base in H is a circle of 1″ radius.

Find the curve of intersection of this cylinder with V, project the cylinder on a plane parallel to the elements to show their true lengths, and develop the cylinder.

Take HV at $y=3.75''$.

For the center of the base in H, $x=2.5''$, $y=6''$.

Let the cylinder lie across the third angle.

To find the true lengths of the elements between the bases in H and V, draw an auxiliary projection of the cylinder on a plane perpendicular to H and parallel to the elements. Take the H trace of this plane crossing HV at $x=6''$.

Pass a right-section plane through the cylinder at a point along the axis 1″ from H; show one-half the section on the auxiliary projection by revolving it on a line which is parallel to the auxiliary plane lying in the section plane and passing through the axis.

For the development, let the right-section line be rectified on a line perpendicular to H V, at $x=10''$.

Let the element which is the shortest between H and the right-section plane be laid out on a horizontal line through the middle of the plate.

12. PLATE IX.

Prob. 23. To develop an oblique cone which is represented in the third angle. Find the intersection of the cone with an hemisphere whose center is at the vertex, and whose radius is 2″. Develop the horizontal projecting cylinder of this intersection, and use this cylinder in developing the cone.

The base of the cone is a circle whose plane is in H, the center at $x=5\frac{1}{2}$, $y=6\frac{1}{4}$, and the radius $1\frac{1}{4}$″.

The vertex is $2\frac{3}{4}$″ from H in V, at $x=4$″.

Short arcs should represent the hemisphere so as not to interfere with other lines of the drawing.

In the development of the projecting cylinder of intersection, take the base of the cylinder perpendicular to H V at $x=\frac{3}{4}$″, and let the longest element coincide with H V.

In the development of the cone, take the vertex at $x=12$″, $y=4\frac{1}{4}$″, and the shortest element in H V.

13. PLATE X.

Prob. 24. Find the intersection of three cylinders, A, B, and C, whose right-sections are circles, and develop each cylinder.

All the cylinders are in the third angle.

The axis of A is vertical, 3″ long and is 2″ from V, at $x=6.25$″. The upper base is in H, and its diameter equals 3″.

Cylinder B is to the left of A and its axis is parallel to HV, 1.5″ from H and 2.5″ from V. The plane of the base is perpendicular to H V, at $x=3.75$″, and the diameter of the base is 2″.

Cylinder C is to the right of A; its axis parallel to V and inclined 30 degrees to H upwards to the right. The axis of C intersects the axis of A at 2.5″ from H. The plane of the base is perpendicular to the axis with its center at $x=8.75$″. The diameter of the base equals 2″.

To develop B let the base rectify on a line perpendicular to H V, at $x=0.75$″; the longest element of the cylinder coinciding with HV.

To develop C let the base rectify on a line perpendicular to H V, at $x=11.75$″; the lowest element coinciding with H V.

14. PLATE XI.

Prob. 25. To develop A of Plate X, let the base be rectified on a line parallel to HV and 1.5″ below it; the element nearest V to be at $x=6.75''$.

15. PLATE XII.

Prob. 26. Find the intersection of a regular, hexagonal prism, A, and a right cylinder, B, with another regular, hexagonal prism, C, and develop each surface.

Prism C has its axis perpendicular to H and parallel to V, two of its sides perpendicular to V, and is 3″ high. The side of the hexagonal base is 1.5″.

In the projection on H, the center of the base is at $x=6.25''$, $y=6.25''$.

The projection on V of the upper base is at $y=4.25''$.

Prism A is parallel to H and makes an angle of 30 degrees with V, on the left of C, its axis intersecting the axis of C. Two of the sides of the prism are parallel to H. The side of the hexagonal base is 1″. The plane of the base is perpendicular to the axis of the prism, the center of the H projection of the base being at $x=3.75''$. The V projection of the axis is at $y=2.5''$.

Cylinder B is parallel to both H and V, on the right of C, the H projection of its axis being at $y=6''$, and the V projection at $y=2.5''$. The radius of the base $=1''$, and its plane is perpendicular to the axis; the H projection of the center of the base being at $x=8.75''$.

To develop A, rectify the hexagon of the base on a line perpendicular to H V, at $x=0.5''$; the middle line of the upper face coinciding with the middle line of the plate.

To develop B, rectify the circumference of its base on a line perpendicular to H V, at $x=12''$, the longest element coinciding with the middle line of the plate.

16. PLATE XIII.

Prob. 27. To develop C of the previous plate, rectify the hexagon of the lower base on a line parallel to H V, at $y=2.5''$, so that the edge nearest V shall be at $x=6.25''$.

17. PLATE XIV.

Prob. 28. Find the intersection of a right cylinder, A, and a regular, hexagonal prism, B, with a regular, hexagonal pyramid, C, and develop each surface.

Pyramid C has its axis perpendicular to H, $3.5''$ long, the H projection at $x=7.25''$, $y=6.5''$. One side of the base is parallel to V. The sides of the base are each $1.5''$ long. The apex of the pyramid is at $4.75''$ in the V projection.

Cylinder A is to the left of C, has its axis parallel to both H and V, and its axis intersects the axis of the pyramid. The V projection of the axis is at $y=2.25''$. The plane of the base is perpendicular to both H and V at $x=5.25''$. The radius of the base $= 0.75''$.

Prism B is in front of C, and has its axis parallel to H and perpendicular to V, the axis intersecting the axis of the pyramid. The V projection of the axis is at $y=3.5''$. The sides of the base are each $0.5''$ long. The plane of the base is parallel to V, and its H projection is at $y=5.5''$.

Draw a profile projection of the three surfaces, the projection of the axis of C being at $x=2.75''$.

Develop A, the base rectifying on a line perpendicular to H V, at $x=11''$, the highest element coinciding with a line at $y=4.25''$.

Develop B, the base rectifying on a line perpendicular to H V, at $x=1.25''$, the middle of the upper face coinciding with a line at $y=6.5''$.

18. PLATE XV.

Prob. 29. Develop C by taking the apex at $x=6.25''$, $y=6''$, and making the edge to the left parallel to V coincide with a line perpendicular to H V, at $x=6.25''$.

19. PLATE XVI.

Prob. 30. Draw the isometric projection (Art. 54, Part I) of the drawing-table whose dimensions are given on the accompanying sketch. The bottom of the drawer is raised $\frac{1}{2}''$ from the bottom of the sides. The front and side pieces are halved together at their joint.

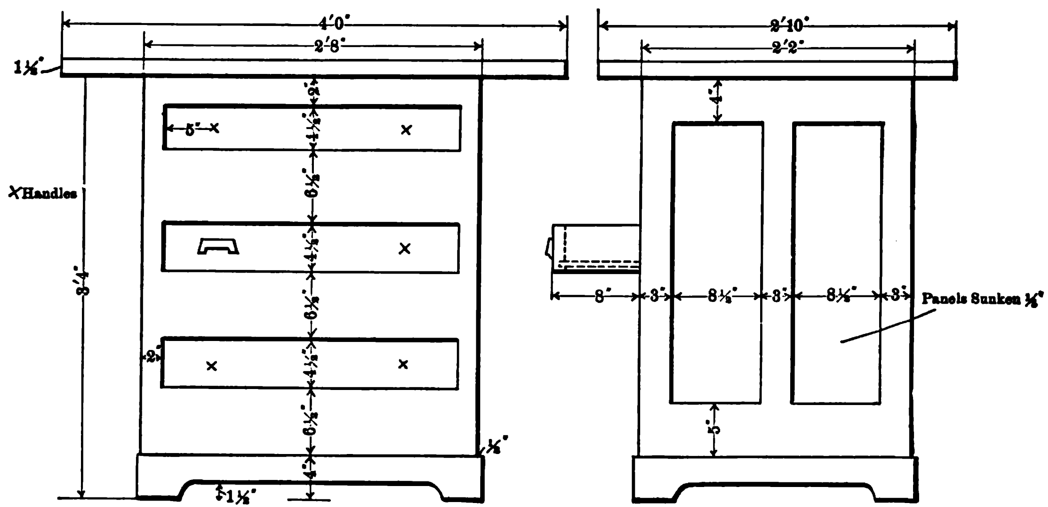


FIG. 1.

Show the second drawer open and projecting 8″. The thickness of the front and sides of the drawer is 1″, that of the bottom $\frac{1}{2}''$.

Let the scale be 1″ to 1′.

Draw the dimension-lines and insert lengths. Letter the title "Drafting-table," above the

drawing, and make below it a scale figured and lettered.

After the drawing is completed, draw the border-line of the usual dimensions, symmetrically placed with relation to the drawing, and cut the plate the usual size.

20. PLATE XVII.

Prob. 31. Draw a pseudo-perspective of the same drawing-table, with dimensions and scale the same. The border is to be placed symmetrically around the drawing as in the previous problem, but not till it is completed.

21. **Shades and Shadows.**—There are no new principles of Descriptive Geometry involved in the subject of shades and shadows. The direction of the parallel rays of light may be that of the body-diagonal of a cube (Art. 34, Part I), or that fulfilling any suitable condition suggested to the draftsman.

Any body upon which the light shines will be divided, by a distinct line or lines, into a light part and a shaded part (Art. 34, Part I). This dividing line is called the *line of shade*, and is the line of tangency of a surface, or surfaces, made up of rays of light tangent to the body. That portion of space from which the light is cut off by this body is said to be the *indefinite shadow* of the body. If any other body is placed within this indefinite shadow, a *definite shadow* will be cast upon it by the first body.

To find the line of shade on a body it is necessary to determine the intersection of the tangent surface of rays with the body. The character of this surface of rays is determined by the form of the body, and may be made up of planes or cylindrical surfaces, or both.

To find the shadow of one body on another it is necessary to find the intersection with the second body of this surface of rays tangent to the first body. Since the tangent surface of rays is wholly made up of planes or cylindrical surfaces, one or both, the problem reduces itself to finding the intersection of planes and cylindrical surfaces with surfaces of all other kinds. And since the tangent surfaces are made up of rays of light, the problem may be further reduced to finding where right lines intersect surfaces of different kinds. (See Plates 33 and 34, Part I.)

22.

PLATE XVIII.

Prob. 32. Draw the projections on H and V of an hexagonal pillar, resting on a square base and surmounted by a square cap-block. Find the shade on the pillar, the shadow of the cap on the pillar, of the pillar on the base, and the shadow of the whole on a horizontal plane coinciding with the lower face of the base. The projection of rays of light on H and V are to make an angle of 45 degrees with H V. The object is to be assumed in the third angle.

The cap is 2″ square and ¼″ thick, with one edge parallel to V.

The base is 2″ square and ½″ thick, with one edge parallel to V.

The pillar is 2″ high between the cap and base, and a side of its base is ¾″.

The planes of two of the faces of the pillar are perpendicular to V.

The axial line of the pillar is projected on H at (5″, 4¼″), and the lower face of the base is at $y = \frac{3}{4}''$.

(After the drawing is completed make a tracing of it for use in a problem to follow later.)

Note: For a more extended course in shades and shadows the problems of Part I may be used.

23. Perspective.—Perspective differs from orthographic projections in that the projecting lines of points *are not parallel*, but *diverge* from some near point which is the position of the eye. The orthographic projections of a point on H and V are required in order to determine its position in space, so that orthographic projections must be used to find perspectives.

The perspective is usually made upon the V plane, which is then called the *picture-plane*.

The orthographic projections of the position of the eye determine what is called the *point of sight*, and the orthographic projections of the line projecting a point on the picture-plane determine what is called a *visual ray;* and the point where this visual ray pierces V (the picture-plane) is called the *perspective of the point* (Art. 63, Part I).

The eye (point of sight) is located in the fourth angle, and the object to be drawn, in the third angle.

Example: In Fig. 2, S is the point of sight, S P a visual ray, and *p*, where the visual ray to the point P pierces the picture-plane, the perspective of the point P.

24. Vanishing-point.—The perspectives of all parallel lines will have a *common point*. This point is called the *vanishing-point* of that system of parallel lines. The vanishing-point of any system of parallel lines may be determined by finding where a parallel line through the point of sight pierces the picture-plane. Because if visual rays are drawn to two different points of any one of these lines, they will determine a plane, called a *visual plane*, the intersection of which with the picture-plane is the *indefinite perspective* of the line. The parallel line and the line through the point of sight parallel to it, determine the same plane; hence where the parallel

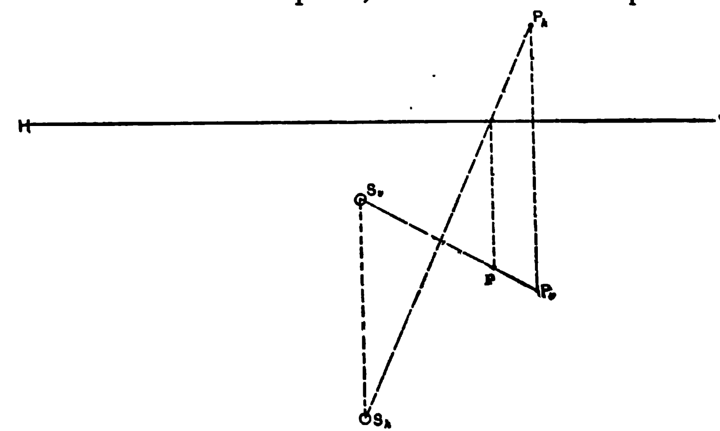


FIG. 2.

line through the point of sight pierces the picture-plane is a point in the indefinite perspective of the line. This parallel line through the point of sight is common to the visual planes of each one of the system of parallel lines whose intersections with the picture-plane determine their indefinite perspectives. Therefore the point in the picture-plane where the line through the point of sight, parallel to the system of parallel lines, pierces it, is a point in the indefinite perspective of each line; ***hence their vanishing-point.***

The vanishing-point of all lines lying in horizontal planes is in the horizon (Art. 63, Part I). A diagonal line is a line lying in a horizontal plane making an angle of 45 degrees with the picture-plane. Its vanishing-point is, therefore, in the horizon at a distance from the vertical projection of the point of sight, *called the center of the picture*, equal to the distance of the point of sight from the picture-plane. (The student should review Arts. 63 and 64, Part I, before attempting the following problems in perspective.)

25. To Find the Vanishing-point of any System of Parallel Lines.—*Having given the center of the picture*, S_v, *the point of distance*, D_v, *and the angles* α *and* β, *that any system of parallel lines makes with* V *and* H, *respectively, to find the vanishing-point of the lines.* It is required to find its distances from a vertical line through the center of the picture, and from the horizon.

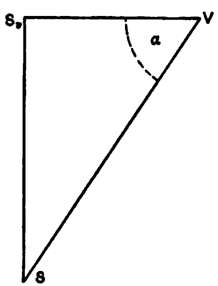


FIG. 3.

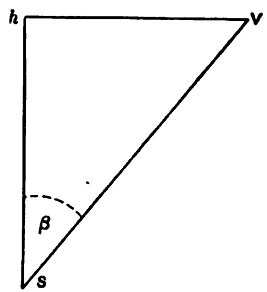


FIG. 4.

Let the triangle $S\,S_v\,V$ (Fig. 3) lie in the vertical projecting plane of a visual ray, S V, passing through the point of sight and parallel to any system of parallel lines; $S\,S_v$ is perpendicular to the picture plane and α is the angle the visual ray makes with V.

In the right triangle $\sin\alpha = \frac{S\,S_v}{S\,V}$, but $S\,S_v = D_v\,S_v$, the distance from the center of the picture to the distance point; therefore

$$S\,V = S_v\,D_v \operatorname{cosec}\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

and

$$S_v V = S_v\,S \cot\alpha = S_v\,D_v \cot\alpha. \quad . \quad . \quad . \qquad (2)$$

Again, let the triangle $S\,h\,V$ (Fig. 4) lie in the horizontal projecting plane of the same visual ray, S V; and let $S\,h$ be a line parallel to H, V the vanishing-point, and h Vthe trace of the projecting plane on the picture-plane. Then β is the angle the visual ray makes with H, and h V the perpendicular distance from the horizon to the vanishing-point.

In this right triangle $\sin\beta = \frac{h\,V}{S\,V}$, or $h\,V = S\,V \sin\beta$.

Substituting the value of S V from (1) in this equation, we have

$$h\,V = S_v\,D_v \operatorname{cosec}\alpha \sin\beta. \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Equation (3) gives the distance, h V, that the vanishing-point is above or below the horizon. Now to find the distance to the right or left of the vertical line through S_v a right triangle may be formed lying in the picture-plane (V) of which S_v V in (2) is the hypothenuse, h V in

(3) one leg, and the other leg the required distance. Calling d the required distance, we have

$$\overline{S_v V}^2 - \overline{h V}^2 = d^2.$$

or substituting the value of S_v V in (2) and h V in (3), we have

$$d = \sqrt{\overline{S_v D_v}^2 \cot^2 \alpha - \overline{S_v D_v}^2 \operatorname{cosec}^2 \alpha \sin^2 \beta}$$

$$= \frac{S_v D_v}{\sin \alpha} \sqrt{\cos^2 \alpha - \sin^2 \beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If $\beta = 0°$, then (3) becomes $h\ V = 0$,

and (4) becomes $d = S_v D_v \cot \alpha$,

that is, the perspectives of all lines lying in horizontal planes, whatever angle they make with V, will vanish in the horizon. If, however, $\alpha = 0°$, the vanishing-point will be in the horizon at infinity.

If $\alpha = 0°$, then (3) becomes $h\ V = \infty$,
and (4) becomes $d = \infty$,

for all values of β, i.e., the vanishing-point of all lines parallel to the picture-plane is at infinity, and the perspectives of the parallel lines of any system of lines parallel to the picture-plane are parallel.

If $\alpha = 90°$, then (2) becomes $S_v\ V = 0$, i.e., the vanishing-point of lines perpendicular to the picture-plane (V) is at S_v.

If $\alpha = 45°$ and $\beta = 0°$, (3) becomes $h\ V = 0$,
and (4) becomes $d = S_v D_v$,

i.e., the vanishing-point of diagonals is at the distance-point.

26. To Find the Vanishing-point of any System of Parallel Lines, Graphically. In Fig. 5, let H V be the line of intersection of H and V, S_v D_v the horizon and S_h A_h

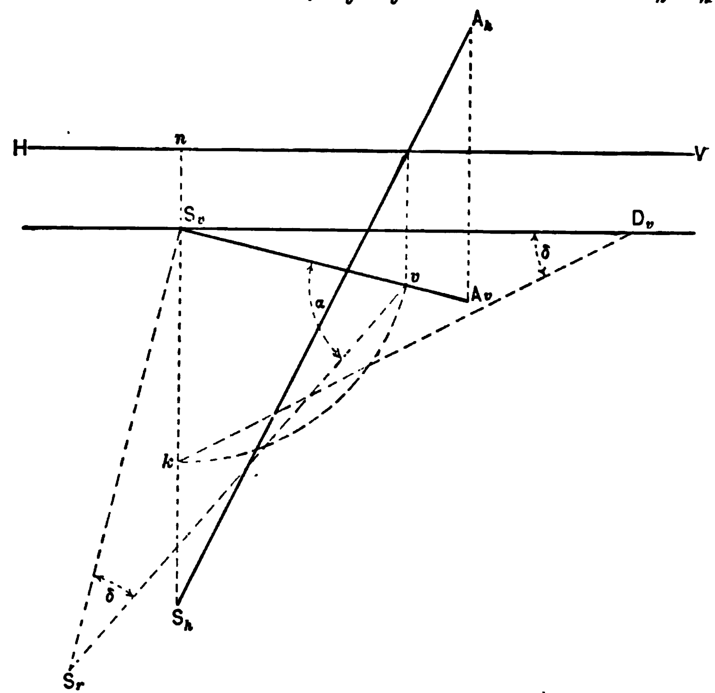


FIG. 5.

—S_v A_v the H and V projections of a line through the point of sight, parallel to a system of paralell lines. By

Art. 23, where the line S A pierces the picture-plane is the vanishing-point, v, of the perspectives of these lines.

By revolving the line S A about the V trace of its V projecting plane, the angle α which the line makes with the picture-plane may be found.

Lay off from $S_v D_v$, at D_v, the angle δ, equal to the complement of α, and find where $D_v k$ meets $S_v S_h$. With $S_v k$ as a radius and S_v as a center draw the arc $k v$, meeting $S_v A_v$ at v, the required vanishing-point. The triangles $S_r S_v v$ and $S_v D_v k$ are equal right triangles; therefore $S_v v = S_v k$.

27. When the horizontal projection of the point of sight, S_h, is not used, as is usually the practice in drawing perspectives, the vanishing-point of any system of parallel lines may be found by the following

Rule: *Form a right triangle on* $S_v D_v$ *as one leg, lay off from* $S_v D_v$ *at* D_v *the complement of the angle that the lines make with V to obtain the direction of the hypotenuse. With the other leg as a radius* ($S_v k$) *and* S_v *as a center describe an arc; where this arc meets the* V *projection of the line through the center of the picture will be the required vanishing-point.*

28. PLATE XIX.

Prob. 33. Draw the perspective of a rectangular, truncated prism, with parallel lines, $\frac{1}{4}''$ apart, on the lateral faces, parallel to the truncated base.

The base of the prism is $2''$ by $1\frac{1}{2}''$; the height of the prism is $3''$. The wider faces make an angle of 30 degrees with V, inclining away from it to the right. The bottom face rests on a horizontal plane at $y=1''$. The upper face is partly truncated by a plane perpendicular to V and making an angle with H of 15 degrees, inclining downwards to the right, beginning on the top base at a point in the middle of the shorter side.

One edge is in the picture-plane at $x=5\frac{1}{4}''$.

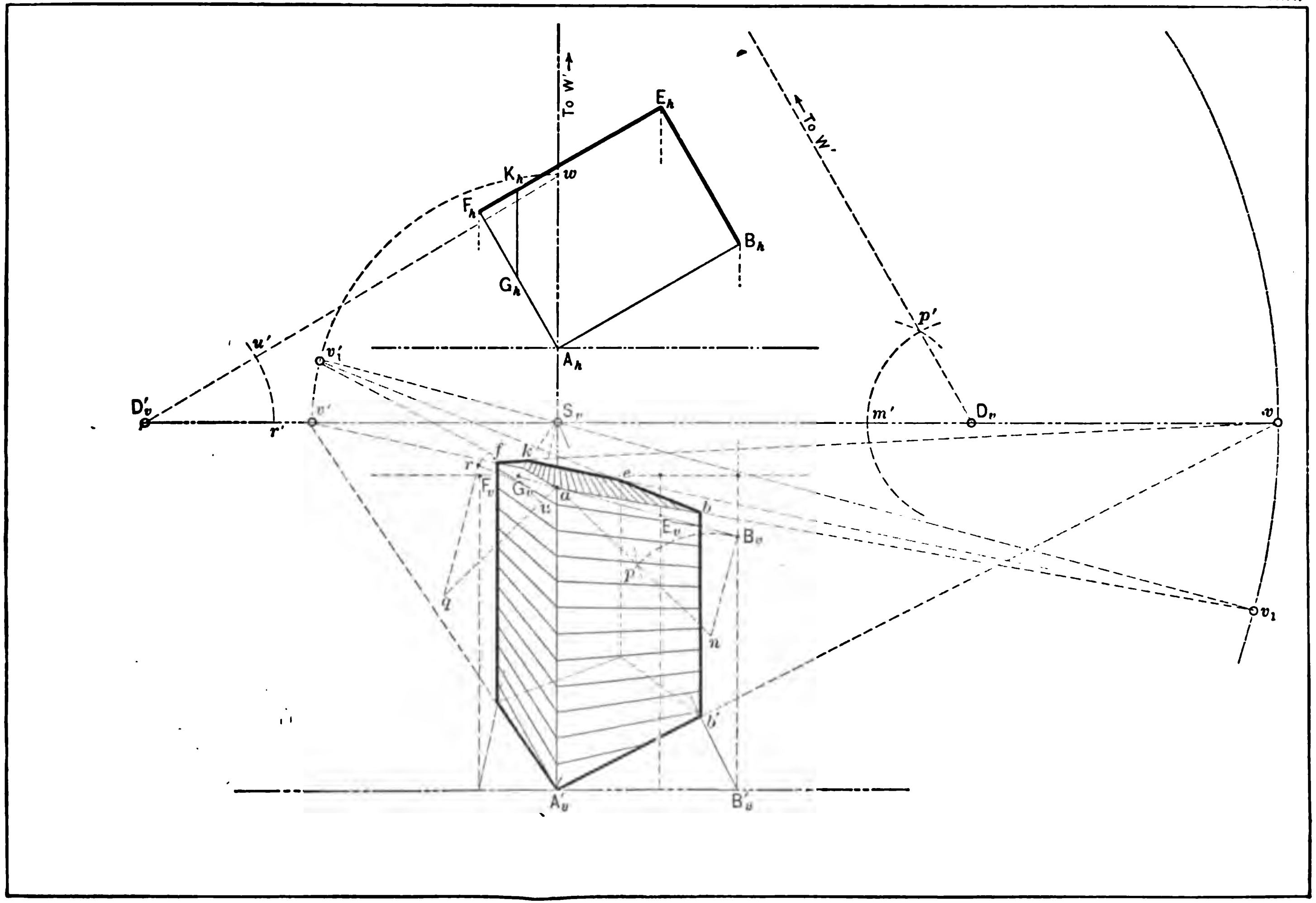
The trace on H of the picture-plane (H V) is at $y=5\frac{1}{4}''$.

The horizon is at $y=4\frac{1}{2}''$.

S_v is at $x=5\frac{1}{4}''$, and D_v is at $x=9\frac{1}{4}''$.

Next find the vanishing-point of the longer side of the lower base, A′ B′ and F′ E′ and the small portion of the upper base parallel to it. Since these are horizontal lines their perspectives will vanish in the horizon. The angle they make with V is 30 degrees. With S_v D_v as one leg of a right triangle and an angle of 60 degrees at D_v construct the triangle S_v D_v w' (w' comes off the plate), then with S_v w' as a radius and S_v as a center describe an arc cutting the horizon in v, the required vanishing-point. (The V projection of the line through S_v parallel to the V projection of A′ B′ coincides with the horizon.) In a similar way v' is found, which is the vanishing-point of the perspectives of the side of the base A′ F′.

To find the vanishing-point of the perspectives of the sides of the base lying in the truncated plane, as A B and K E: First determine the angle these lines make with V, by the method of determining the angle a line makes with a plane of projection as shown in the triangle, a B_v n. Also find its complement, p n B_v. Through S_v draw a line, S_v v_1, parallel to the vertical projection of A B, which is the line a B_v. With S_v D_v as one leg of a right triangle and the angle at D_v equal to p n B_v construct the triangle S_v D_v w'; then with S_v w' as a radius and S_v as a center describe the arc cutting the line S_v v_1 in v_1, the required vanishing-point. Similarly v_1', the vanishing-point of the perspectives of E B and A G, may be found. The perspective of F G vanishes at v', and that of F K at v. The perspectives of the parallel lines on the face A B A′ B′, which are parallel to the truncated face, vanish at v_1, and those on the face A F A′ F′ vanish at v_1'. The section-lines on the truncated base are perpendicular to V and vanish at S_v.


To W'
To W'
E_h
K_h
w
F_h
B_h
G_h
A_h
p'
u'
v_1'
D_v'
r'
v'
S_v
m'
D_v
v
v_1
f
k
e
r
F_v
G_v
a
b
E_v
B_v
p
q
n
b'
A_v'
B_v'

29. PLATE XX.

Prob. 34. Find the perspective of the body in Prob. 32, Plate XVIII, and the perspectives of its shades and shadows.

Let H V be at $y=5''$.

The horizon at $y=4''$.

S_v at $x=7''$, and

D_v at $x=2''$.

The front face of the base is taken in the picture-plane, the object being in the third angle.

Use the tracing made of the problem referred to instead of drawing again the projections.

Find the vanishing-point of the perspectives of the shadows of vertical edges; it should fall at D_v.

Where there are only a few lines that are parallel in a perspective, it is often simpler to find the perspectives of the extremities of the lines than to attempt to find the vanishing-point of their perspectives.

In this problem, the shadow having already been found, the perspective of the shadow may be determined directly by finding the perspectives of the outline, but in general the perspective of the shadow is found without first finding the shadow, but rather by finding where the perspective of a ray of light through a point meets the perspective of its projection on the plane or surface upon which the shadow of the point s cast (see the method of Part I).

30. PLATE XXI.

Prob. 35. Find the perspective of the same body, substituting a cylinder for the hexagonal pillar.

The radius of the base to be $\frac{3}{4}''$.

Other problems in perspective may be added by using the problems in Part I.

[illegible]